Explaining output volatility: The case of taxation

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Abstract

This paper presents strong empirical evidence that the observed heterogeneity of output volatility across countries and over time is partly endogenous. In particular, based on a closed-form solution we obtain a (long-run) equilibrium relationship between taxes and output volatility in the stochastic neoclassical model by showing that asymptotically the variance of output growth rates is affected by the *level* of taxes, without affecting the mean. We estimate the tax semi-elasticities on output volatility and provide convincing empirical evidence that taxes are important to understand differences in output volatility among OECD countries.

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1 Introduction

This paper presents strong empirical evidence that the observed heterogeneity of output volatility across countries and over time is partly endogenous. Most studies investigating why output growth has become less volatile in the US and in many other OECD countries (among others McConnel and Perez-Quiros 2000, Stock and Watson 2002, 2005) explain the empirical observation with changes in inventory management, improved central bank policy, changes in demographics and/or less volatile labor input, but also leave a large fraction unexplained.[1] Surprisingly little attention has been given to fiscal policy.[2]

Since most of these studies do not inquire the structural channels through which output volatility is affected, an open question is which role fiscal policy, in particular tax policy, has on the propagation of exogenous shocks. Although many dynamic stochastic general equilibrium (DSGE) models consider the variance of the innovations to technology as exogenous, the way how these shocks are propagated through the economy is based on optimization principles and thus subject to changes due to distortionary taxation.[3]

Our empirical motivation stems from the fact that major US tax reforms took place around the point in time where the break in output volatility is usually identified.[4] In this period the focus of US policy debates was on the Economic Recovery Tax Act (ERTA), the first of the Reagan tax cuts (also known as Kemp-Roth Tax Cut) with large economic effects (Auerbach and Slemrod 1997). Similarly, the moderation of output volatility in the UK was accompanied by massive tax cuts (Giles and Johnson 1995).[5]

In a nutshell, we show that the standard measure of output volatility is affected by the *level* of taxes and provide convincing empirical evidence that taxes are important to understand differences in output volatility among OECD countries.

Our framework builds on Posch and Wälde (2009), who obtain closed-form measures in a model where output volatility is fully endogenous and identify three channels through which taxes can have effects on output volatility. This paper now thoroughly studies the propagation component and estimates the tax effects empirically. For this objective we use a continuous-time formulation which allows us to analytically derive tax effects on output volatility under Normal uncertainty.[6] As it turns out, second-order approximation

[1]Stock and Watson (2002) surveys the "great moderation" literature. Recent work includes Cecchetti et al. (2006), Justiniano and Primiceri (2008), Jaimovich and Siu (2009), Stiroh (2009), Canova (2009).

[2]There is a tradition studying the impact that fiscal policy has on macroaggregates (see e.g. Brock and Turnovsky 1981, Becker 1985, Danthine and Donaldson 1985, Greenwood and Huffman 1991).

[3]This is in the tradition of Brock and Mirman (1972), Kydland and Prescott (1982), and Long and Plosser (1983). If aggregate shocks are endogenous (e.g. Bental and Peled 1996, Matsuyama 1999, Francois and Lloyd-Ellis 2003, 2008, Wälde 2005) taxes could even directly affect output volatility.

[4]The break for the US is often dated in 1984Q1 or 1984Q2 (Kim and Nelson 1999, McConnel and Perez-Quiros 2000) while other estimates range from 1982Q4 to 1985Q3 (Stock and Watson 2002).

[5]Cecchetti et al. (2006) identify two breaks in 1981Q2 and 1991Q4 for the UK.

[6]An introduction to continuous-time DSGE models can be found in Turnovsky (2000). A comparison

schemes are necessary in order to uncover tax effects on output volatility in the standard discrete-time DSGE model (Schmitt-Grohé and Uribe 2004), which makes analytical attempts more challenging in the traditional formulation.[7]

The contribution of this paper is primarily empirical. One of the most surprising findings is the strong empirical link between taxes and output volatility. Using a panel of OECD countries from 1970 to 2009, our results demonstrate that the effects of taxes are robust and of economic relevance. Although the highlighted channel does not fully account for the size of the observed effects, the qualitative empirical results support our theoretical model: the Mendoza et al.'s (1994) tax ratios on labor and corporate income are negatively correlated with output growth variance, while the capital tax ratio has a positive effect. For the consumption tax ratio we do not find a robust link. Our results are found to be robust to the estimation strategy (regression-based or likelihood-based), and to different specifications for the controls and volatility measures.

From a theoretical perspective, we show that in the stochastic neoclassical model taxes affect the second moment rather than the mean of output growth rates. Based on a closed-form expansion for the variance of output growth rates, we find a negative link between output volatility and the income/investment tax, a positive link with the tax on wealth, while the consumption tax has no effect. The intuition is that output volatility can be decomposed into the variance of the exogenous shocks and an endogenous component that is governed by fundamentals (henceforth propagation component). Taxes affect the propagation component by distorting the consumption-savings decision, which in the stochastic neoclassical model affects the variability of capital rewards and eventually translates into changes in output volatility. This link establishes our asymptotic result of a (long-run) equilibrium relationship between taxes and output volatility.

There is a related literature on macro volatility. Much of it focuses on less developed countries and financial development (Denizer et al. 2002, Lensink and Scholtens 2004) or institutions (Acemoglu et al. 2003). Our estimates confirm a link between the variance of output growth and standard controls in volatility regressions including the mean output growth, the variability of real effective exchange rates, and various variables capturing fiscal and monetary policy. We do not find statistically significant effects, however, for measures of financial development and openness for our sample of OECD countries.

In the paper we proceed as follows. Section 2 derives a closed-form expression for the tax effects on the mean and variance of output growth rates in the stochastic neoclassical model. Section 3 describes data and our estimation strategy. Section 4 presents our main empirical results and further robustness results. Section 5 concludes.

between different production technologies in stochastic continuous-time models is Wälde (2011).

[7] Note that numerical solutions of an equivalent discrete-time formulation give very similar results.

2 Taxes and output volatility

This section provides a theoretical model for tax effects on output volatility. In its structure it describes a (continuous-time) real business cycle model with a government sector. For this economy we obtain a closed-form measure of output volatility based on growth rates. In that view, we illustrate *one* channel through which taxes affect macro volatility instead of aiming to match the observed empirical effects quantitatively.[8]

2.1 The model

As the technological setup of the economy is that of a standard real business cycle model, we keep the outline of the model brief. The introduction of government activities and the implications for household behavior will be presented in more detail.

Production possibilities. The single good is produced according to a Cobb-Douglas function,

$$Y_t = A_t K_t^{\alpha} L^{1-\alpha}, \tag{1}$$

where L is total constant labor supply, K_t is the aggregate capital stock. Uncertainty enters through an geometric process for total factor productivity, A_t, driven by a standard Brownian motion B_t,

$$dA_t = \mu A_t dt + \eta A_t dB_t. \tag{2}$$

Output Y_t is used for producing consumption goods C_t and investment goods I_t. The aggregate capital stock increases if gross investment I_t exceeds depreciation δK_t,

$$dK_t = (I_t - \delta K_t)dt. \tag{3}$$

Government. The government exogenously levies taxes on income, τ_i, on wealth, τ_a, on consumption expenditures, τ_c, and on investment expenditures, τ_k. It cannot save or run debt and uses all revenues to provide basic government services G,

$$G_t = \tau_i(Y_t - \delta K_t) + \tau_k(I_t - \delta K_t) + \tau_c C_t + \tau_a(1 + \tau_k)K_t \geq 0. \tag{4}$$

We assume a myopic government simply providing (exogenous) public goods without interest in neither stabilization policy nor optimal taxation, where the tax policy is under control of the government.[9] In order to illustrate the incentive effects of distortionary taxation on output growth volatility in an otherwise frictionless economy we abstract from

[8] Alternatively, to study the quantitative tax effects numerically one may use a state-of-the-art DSGE model (Fernández-Villaverde and Rubio-Ramírez 2007, Justiniano and Primiceri 2008).

[9] Our causal relationship runs from taxes to output volatility. There is mixed empirical evidence on whether taxes are endogenous. Tax cuts are typically the policy response to specific economic conditions. As we show below, a change in the tax structure has effects on output volatility. Whether or not this policy change was triggered by some exogenous event is beyond the scope of this paper.

the possibility of debt. One could also interpret the taxes as wedges between competitive prices and observed prices (see Chari et al. 2007).

Preferences. The economy has a large number of representative households, who maximize expected utility, given by the integral over utility, $u = u(c_t)$, resulting from consumption flows, c_t, discounted at the subjective rate of time preference, ρ,

$$U_0 = E_0 \int_0^\infty e^{-\rho t} u(c_t) dt, \quad u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \tag{5}$$

where instantaneous utility is characterized by constant relative risk aversion.

The budget constraint of the representative household reads

$$da_t = \left(\left(\frac{1-\tau_i}{1+\tau_k}(r_t - \delta) - \tau_a \right) a_t + \frac{1-\tau_i}{1+\tau_c} w_t - c_t \right) dt, \tag{6}$$

where w_t denotes the real wage rate, and r_t the rental rate of capital, both before tax.

Equilibrium properties. In equilibrium, factors are rewarded by $w_t = Y_L$, and $r_t = Y_K$ (marginal products), respectively. Market clearing demands $Y_t = C_t + I_t + G_t$, or

$$(1-\tau_i)Y_t \;\; = \;\; (1+\tau_c)C_t + (1+\tau_k)I_t + (1+\tau_k)\left(\tau_a - \frac{\tau_i + \tau_k}{1+\tau_k}\delta \right) K_t. \tag{7}$$

where we inserted the government budget constraint (4). Note that the quantities C_t and I_t are after taxation. Since markets are perfectly competitive, the producer price of the production, consumption, and investment good will be identical,

$$p_t^Y = p_t^C = p_t^K. \tag{8}$$

When consumption and investment goods are sold, they are taxed differently such that consumer prices are $(1+\tau_c)\, p_t^C$ and $(1+\tau_k)\, p_t^K$. In order to rule out arbitrage between different types of goods, we assume that a unit of production is useless for other purposes once it is assigned for a specific purpose.

Solving the model requires the first-order condition for consumption, the aggregate constraints (2) and (3), the goods market equilibrium (7), and optimality conditions of perfectly competitive firms. Thus we obtain a system of equations determining, given boundary conditions, the time paths of macro aggregates and factor rewards.

In order to obtain tax effects on macro volatility we may follow two approaches. First, we may use higher-order approximation schemes to study the effects of distortionary taxes on macro dynamics and thus on macro volatility numerically. Second, we may obtain the tax effects on macro volatility analytically. In this paper we follow the latter approach and use parametric restrictions under which the model has explicit solutions. The main advantage is the availability of closed-form expressions for the asymptotic distributions of macro aggregates and measures of output volatility. We believe that this structural approach allows us best to understand the nature of the tax effects.

2.2 Explicit solutions

Though our economy is set up in continuous time, we easily obtain output growth rates as observed in the data: use Itô's formula to compute the differential of logarithmic output,

$$\begin{aligned} dY_t &= (\mu + \alpha(I_t/K - \delta))\, Y_t dt + \eta Y_t dB_t \\ \Leftrightarrow \; d\ln Y_t &= \left(\mu - \tfrac{1}{2}\eta^2 + \frac{1-\tau_i}{1+\tau_k} r_t - \alpha \frac{1+\tau_c}{1+\tau_k} C_t/K_t - \alpha\left(\tau_a - \frac{1-\tau_i}{1+\tau_k}\delta\right)\right) dt + \eta dB_t, \end{aligned}$$

in which we inserted $r_t = \alpha Y_t/K_t$ and gross investment

$$I_t = \frac{1-\tau_i}{1+\tau_k} Y_t - \frac{1+\tau_c}{1+\tau_k} C_t - \left(\tau_a - \frac{\tau_i+\tau_k}{1+\tau_k}\delta\right) K_t. \tag{9}$$

Now integrate over the observation period to get output growth rates at frequency Δ, $\Delta y_t \equiv \ln Y_t - \ln Y_{t-\Delta}$.[10] The infinitesimal change $d\ln Y_t$ technically describes a controlled stochastic differential equation (SDE), i.e., controlled by the yet unknown households' consumption choice. In general equilibrium this choice determines investment and thus the process for the capital stock and future capital rewards. Hence, the exogenous shocks enter output growth rates both contemporaneously and inter-temporally because they are propagated through capital accumulation (propagation mechanism).

Theorem 2.1 *If $\alpha = \sigma$, where α is the output elasticity of capital and σ the parameter of relative risk aversion, consumption is linear in the capital stock, $C_t = \frac{1+\tau_k}{1+\tau_c}\phi K_t$,*[11]

$$\phi = \frac{\rho}{\sigma} + \frac{1-\sigma}{\sigma}\left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right). \tag{10}$$

Proof. Appendix A.2.1. ■

Corollary 2.2 *The rental rate of capital (capital rewards) obeys*

$$dr_t = c_1 r_t\,(c_2 - r_t)\,dt + \eta r_t dB_t, \tag{11}$$

where $c_1 \equiv \frac{1-\alpha}{\alpha}\frac{1-\tau_i}{1+\tau_k}$, *and* $c_2 \equiv \frac{1+\tau_k}{1-\tau_i}\left(\frac{\alpha}{1-\alpha}\mu + \rho + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)$.

The SDE in (11) is a geometric reverting diffusion process known as the stochastic Verhulst equation (Sørensen 1991). Accordingly, c_2 defines the non-stochastic steady state to which r_t tends, and c_1 is the speed of reversion. It can be shown that r_t has a limiting Gamma distribution where all moments are available in closed form (Merton 1975). Since our distribution now is a function of taxes, this result is crucial to understand the tax effects on empirical measures of volatility such as the variance of output growth rates.

[10] Our convention is to use $\Delta = 1/12$ for monthly data, $\Delta = 1/4$ for quarterly data, $\Delta = 1$ for annual data. Thus all parameters are measured at the annual basis (base unit of time is years).

[11] This parametric restriction is fairly well established in macroeconomics (Chang 1988, Xie 1991, 1994, Boucekkine and Tamarit 2004, Wälde 2005, Smith 2007, Posch 2009).

Corollary 2.3 *The growth rate of output Δy_t reads*

$$\Delta y_t \;=\; \left(\mu - \tfrac{1}{2}\eta^2\right)\Delta + \sigma\Delta c_t + \eta(B_t - B_{t-\Delta}), \qquad (12)$$

in which $\Delta c_t \equiv \ln C_t - \ln C_{t-\Delta}$ denotes the consumption growth rate at frequency Δ.

Corollary 2.3 is remarkable as it implies a linear relation between output growth rates and consumption growth rates, which clearly depends on our restriction $\alpha = \sigma$.[12] Our solution (12) will be used to both illustrate and estimate tax effects on macro volatility through the propagation mechanism. In a nutshell, distortionary taxes affect the variance of capital rewards r_t by changing the dynamics in (11). Changes in the variance of $\frac{1-\tau_i}{1+\tau_k}r_t$ propagate further to the variance macro aggregates. The intuition behind this result is that the Euler equation relates consumption growth rates to (after-tax) capital rewards. In particular, the proof to Corollary 2.3 shows that our solution implies

$$\sigma\Delta c_t = \frac{1-\tau_i}{1+\tau_k}\int_{t-\Delta}^{t} r_s ds - \left(\rho + \tau_a + \frac{1-\tau_i}{1+\tau_k}\delta\right)\Delta. \qquad (13)$$

It can be interpreted as the discrete-time version of the Euler equation for $\alpha = \sigma$.

2.3 Tax effects in the stochastic neoclassical model

We are now prepared to study the tax effects on output growth rates. Clearly, taxes affect growth rates in the short run directly by affecting the disposable income and indirectly via capital accumulation, which has been widely discussed in the growth literature. In order to derive effects of taxation on moments of the distribution of growth rates we focus on the long-run effects or the unconditional moments.

Using output growth rates in (12), it can be easily shown that (cf. Appendix A.3)

$$E(\Delta y_t) \;\equiv\; \lim_{t\to\infty} E_0(\Delta y_t) = \tfrac{1}{1-\alpha}\left(\mu - \tfrac{1}{2}\eta^2\right)\Delta, \qquad (14)$$

$$Var(\Delta y_t) \;\equiv\; \lim_{t\to\infty} Var_0\left(\Delta c_t\right)\sigma^2 + \eta^2\Delta. \qquad (15)$$

Intuitively we obtain these results for two reasons. First, the mean does not depend on taxes because it is only driven by the exogenous TFP process. Both consumption and output asymptotically grow at the same expected rate.[13] Second, since optimal consumption is a linear function of the capital stock - a deterministic process - consumption growth rates and the Brownian motion increments in (12) are uncorrelated.

Unfortunately, the variance of consumption growth rate is more involved because it refers to the variance of the integrated process of capital rewards in (13). Because of

[12] The key mechanism is that consumption becomes linear in the capital stock, which determines capital rewards in (11). Another solution with similar dynamics for capital rewards is provided in the appendix.

[13] The variance of consumption growth will be more sensitive to taxes than output growth since $\sigma < 1$.

Table 1: Tax effects in the stochastic neoclassical model

	Marginal tax effects			
	τ_i (income)	τ_c (cons.)	τ_k (investment)	τ_a (wealth)
$E(\Delta y_t)$	0	0	0	0
$Var(\Delta y_t)$	$-\frac{\alpha}{1-\alpha}\frac{1}{1+\tau_k}\delta\frac{1}{2}\eta^2\Delta^2$	0	$-\frac{\alpha}{1-\alpha}\frac{1-\tau_i}{(1+\tau_k)^2}\delta\frac{1}{2}\eta^2\Delta^2$	$\frac{\alpha}{1-\alpha}\frac{1}{2}\eta^2\Delta^2$

Notes: This table reports the marginal tax effects of time-invariant tax rates on volatility and growth in the neoclassical model. We present the mean and variance of output growth per unit of time Δ, neglecting third-order terms.

the non-linear dynamics in (11) no analytical expression exist for the variance.[14] Using closed-form expansions, the variance of consumption growth is (see Appendix A.3)

$$\lim_{t\to\infty} Var_0(\Delta c_t) = \left(c_1c_2 - \tfrac{1}{2}\eta^2\right)\left(\tfrac{1}{1-\alpha}\right)^2 \tfrac{1}{2}\eta^2\Delta^2 + O(\Delta^3). \tag{16}$$

Neglecting third-order terms and using $\alpha = \sigma$, we obtain

$$\begin{aligned} Var(\Delta y_t) &\approx \eta^2\Delta + \left(c_1c_2 - \tfrac{1}{2}\eta^2\right)\left(\tfrac{\alpha}{1-\alpha}\right)^2 \tfrac{1}{2}\eta^2\Delta^2 \\ &= \eta^2\Delta + \tfrac{\alpha}{1-\alpha}\left(\tfrac{\alpha}{1-\alpha}\mu + \rho + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a - \tfrac{\alpha}{1-\alpha}\tfrac{1}{2}\eta^2\right)\tfrac{1}{2}\eta^2\Delta^2. \end{aligned} \tag{17}$$

It shows that second-order approximations are necessary in order to capture tax effects on macro volatility, which linearization methods would neglect. Table 2 illustrates that these second-order effects are economically important: a plausible scenario for the UK is that the tax on wealth decreased by 5 percentage points from the 1980s to the 1990s (as we discuss below in Section 3.1). Such tax change implies a decline in output growth variance of 6.3%.[15] To put these numbers into perspective, the UK experienced a decline in output growth variance by 22.6% over the same time period. These results confirm that higher-order approximation schemes are essential tools to capture potentially important economic effects in DSGE models (Schmitt-Grohé and Uribe 2004).

Economically, a higher tax on wealth, τ_a, clearly distorts the consumption-saving decision. Incentives for investment relative to consumption will be lower since the returns from holding capital decrease, similar to a higher rate of depreciation, δ. Individuals prefer consumption today instead of deferring it to the future. The non-stochastic steady state for capital rewards, c_2, increases (effective capital stock decreases) as from (9) less resources are used for investment. Since the innovations are proportional to r_t in (11),

[14] For a similar mean-reverting model, the Ornstein-Uhlenbeck process $dr_t = c_1(c_2 - r_t)dt + c_3 dB_t$, the auto-covariance function and thus the variance of the integrated process is available analytically. In this case, $Var(\int_{t-\Delta}^t r_s ds)$ is proportional to $Var(r) \equiv \lim_{t\to\infty} Var_0(r_t)$, and $Var(\int_{t-\Delta}^t r_s ds) \approx Var(r)\Delta^2$ coincides with the second-order Taylor approximation about $\Delta = 0$.

[15] It is notable that the implied change of the variance of (before-tax) capital rewards is -37.6%.

Table 2: Calibrated tax semi-elasticities and a plausible tax scenario

	(a) Calibration	(b) Tax semi-elasticities				(c) Tax reform	
	no taxes	τ_i (income)	τ_c (cons.)	τ_k (investment)	τ_a (wealth)	baseline	effect
$E(\Delta y_t)$	0.0188	0	0	0	0	0.0188	0
$Var(\Delta y_t)$	0.0727	-0.10%	0	-0.10%	$+1.29\%$	0.0750	-7.04%
$E(r_t)$	0.1091	$+0.32\%$	0	$+0.31\%$	$+9.17\%$	0.2043	-25.06%
$Var(r_t)$	0.0102	$+1.33\%$	0	$+1.32\%$	$+9.17\%$	0.0295	-2.58%

Notes: This table illustrates tax effects on macro variables for technology parameters $(\rho, \alpha, \sigma, \delta) = (0.02, 0.75, 0.75, 0.075)$, other parameters $(\mu, \eta) = (0.005, 0.025)$, and taxes $(\tau_i, \tau_c, \tau_k, \tau_a) = (0, 0, 0, 0)$. It shows (a) the implied mean and variance (multiplied by a factor 100) of annual output growth rates, and the mean and variance (multiplied by 100) of before-tax capital rewards, (b) percentage changes due to a one percentage point increase in the tax rate, and (c) a tax scenario: the baseline calibration is $(\tau_i, \tau_c, \tau_k, \tau_a) = (0.35, 0.1, 0, 0.05)$, the post-reform values are $(\tau_i, \tau_c, \tau_k, \tau_a) = (0.5, 0.1, 0, 0.005)$.

this in turn results into a higher variance of r_t, the variance of consumption growth rates in (16), and finally translates into higher output volatility in (17).

The consumption tax, τ_c, reduces the level of consumption but does not affect the level of investment. The consumption tax therefore acts like a lump sum tax and does neither affect the non-stochastic steady-state nor other moments of capital rewards with no effects on the variance of consumption and output growth rates.

In order to understand the effects of other taxes, consider at first the case where $\delta = 0$. If there was no depreciation, our solution implies that a higher income tax, τ_i, reduces disposable income and thus the level of investment. The non-stochastic steady state for (before-tax) capital rewards, c_2, increases whereas the speed of reversion, c_1, decreases. Similar effects are implied by a higher investment tax, τ_k, which makes consumption today relatively more attractive. Both effects, however, would not translate into a higher variance of consumption growth rates since after-tax capital rewards, $\frac{1-\tau_i}{1+\tau_k} r_t$, are not affected in both cases. For $\delta > 0$, however, we observe effects on macro volatility for the following reason. Since only *net* capital returns are taxes, the implied refund scheme decreases the effective rate of depreciation, $\frac{1-\tau_i}{1+\tau_k}\delta$. Individuals prefer to defer consumption to the future, the non-stochastic steady state for capital rewards, c_2, decreases. This in turn implies asymptotically smaller innovations in (11), lower variance of consumption growth rates in (16), and finally translates into lower output volatility in (17).

The bottom line is that tax effects on the variance of (after-tax) capital rewards translate to tax effects on the variance of consumption and output growth rates. It turns out that the asymptotic moments of capital rewards are quite sensitive to taxes.

2.4 Taking the model to the data

After having studied the qualitative and quantitative effects of taxes on output volatility, we now ask whether such effects can be found empirically. To this end we define the variance $h \equiv Var(\Delta y_t) = f(\tau_i, \tau_k, \tau_a)$ and log-linearize (17) around $\bar{h} = f(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)$,[16]

$$\log h = \log e^{\log h} = \log g(\log h) \quad \approx \quad \log \bar{h} + f(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)^{-1}(\log h - \log \bar{h}).$$

Similarly, for the right-hand side we obtain

$$\log h \quad \approx \quad \log \bar{h} + \frac{f_{\tau_i}(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)}{f(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)}(\tau_i - \bar{\tau}_i) + \frac{f_{\tau_k}(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)}{f(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)}(\tau_k - \bar{\tau}_k) + \frac{f_{\tau_a}(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)}{f(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)}(\tau_a - \bar{\tau}_a).$$

Equating both sides, we finally obtain our testable empirical implications as

$$\begin{aligned} \log h \quad &\approx \quad \log \bar{h} + f_{\tau_i}(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)(\tau_i - \bar{\tau}_i) + f_{\tau_k}(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)(\tau_k - \bar{\tau}_k) + f_{\tau_a}(\bar{\tau}_i, \bar{\tau}_k, \bar{\tau}_a)(\tau_a - \bar{\tau}_a) \\ &\equiv \quad \beta_0 + \beta_1 \tau_i + \beta_2 \tau_k + \beta_3 \tau_a, \end{aligned} \tag{18}$$

in which we define semi-elasticities $\beta_1 \equiv -\frac{\alpha}{1-\alpha}\frac{1}{1+\bar{\tau}_k}\delta\frac{1}{2}\eta^2\Delta^2$, $\beta_2 \equiv -\frac{\alpha}{1-\alpha}\frac{1-\bar{\tau}_i}{(1+\bar{\tau}_k)^2}\delta\frac{1}{2}\eta^2\Delta^2$ and $\beta_3 \equiv \frac{\alpha}{1-\alpha}\frac{1}{2}\eta^2\Delta^2$ (cf. Tables 1 and 2), i.e., the percentage change in output growth variance if tax rates are changed by 1 percentage point. Using the theoretical model to formulate hypotheses, we would expect to find $\beta_1, \beta_2 < 0$ and $\beta_3 > 0$.

Our empirical strategy is to make use of the heterogeneity of volatility and taxes within two frameworks. The first approach is regression-based and uses (18) to estimate tax effects on output volatility (Section 3.2.1), whereas the second approach treats volatility as a latent variable (Section 3.2.2). For the latter approach, we use equation (12) and define a new variable $\epsilon_t \equiv (\mu - \frac{1}{2}\eta^2)\Delta + \sigma\Delta c_t + \eta(B_t - B_{t-\Delta}) - \theta$ in which $\theta \equiv E(\Delta y_t)$ together with (18) and estimate the following model of (conditional) heteroscedasticity,

$$\Delta y_t \quad = \quad \theta + \epsilon_t, \quad \text{where} \quad E(\epsilon_t) = 0,\ Var(\epsilon_t) = h, \tag{19a}$$

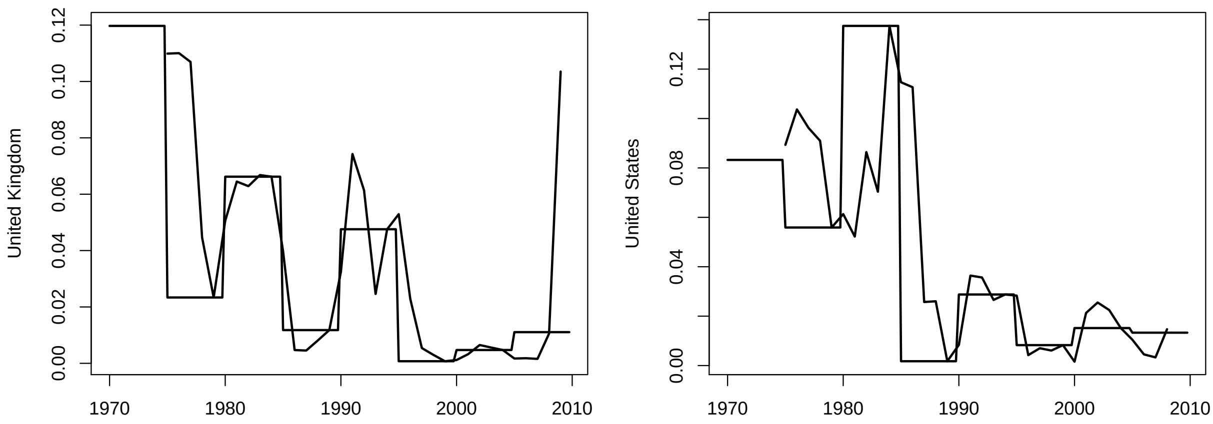
$$\log h \quad = \quad \beta_0 + \beta_1 \tau_i + \beta_2 \tau_k + \beta_3 \tau_a. \tag{19b}$$

We exploit the fact that ϵ_t is a residual term with zero mean and tax-dependent variance. It denotes the deviation of the observed growth rate from its long-run mean, capturing all the transitional dynamics of the neoclassical model. If capital rewards are above average (technically if the capital stock is below the mean of its stationary distribution), the growth rate tends to be higher than its long-run mean.

There are two important caveats. First, equation (18) is an asymptotic result. Thus our empirical analysis should emphasize the cross-sectional relative to the time dimension. For this reason, our benchmark specification neglects other controls in (19a) in order to

[16] A Taylor approximation of $f(x, y) = g(x, \log y)$ for the case where y is strictly positive around $\bar{x}$ and $\bar{y}$ gives $g(x, \log y) \approx g(\bar{x}, \log \bar{y}) + g_x(\bar{x}, \log \bar{y})(x - \bar{x}) + g_{\log y}(\bar{x}, \log \bar{y})(\log y - \log \bar{y})$.

Figure 1: Comparison of different *observed* volatility measures



Notes: These figures illustrate two *observed* volatility measures for the UK (left panel) and the US (right panel) starting in 1970. The first measure results from a fixed-window (five-years) approach gathering the period specific variance of annual growth rates of real GDP per capita (in percent). The second measure is based on five-year rolling window.

focus on the asymptotic properties of our model in which the second rather than the first moment depends on taxes.[17] Second, theoretical marginal tax rates are not observable, which complicates a structural interpretation of our estimates. For example, our effective tax on capital measures the overall tax burden on capital, including a combination of income tax and taxes on wealth - in fact, a linear combination of β_1 and β_3.

3 Data and estimation strategy

Our approach to study output volatility patterns is to use a balanced panel of OECD countries[18] spanning the years from 1970 to 2009. This provides the possibility of learning about the behavior of a single country by observing the behavior of other countries.

3.1 Data

In what follows we use measures of volatility and the effective tax burden at the macro level. The standard measure of macro volatility is the variance (or standard deviation) of output growth rates. Our focus is on the variance of annual growth rates of real GDP per capita.[19] The reason for using annual data is the availability of tax measures.

[17] We also include two lags of growth rates and taxes in (19a) and account for potentially non-stationary variables in (19b) when analyzing robustness of our empirical results in Section 4.2.

[18] Most of the empirical results are based on a sample of 20 OECD countries, i.e., Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Korea, the Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, United Kingdom, United States.

[19] Robustness checks (not reported) based on quarter-to-quarter growth rates, on four-quarter rolling growth rates, as well as on HP-filtered cyclical components give similar results.

Table 3: Linking theoretical tax rates to tax ratios

	income tax, τ_i	consumption tax, τ_c	investment tax, τ_k	tax on wealth, τ_a
$LABOR$	$\times$			
$CAPITAL$	$\times$		$\times$	$\times$
$CORP$	$\times$			
$CONS$		$\times$		

Notes: Based on Mendoza et al.'s (1994) definitions, $LABOR$ denotes the labor income tax ratio, $CAPITAL$ is the capital tax ratio (including taxes on property), $CORP$ is the corporate income tax ratio, $CONS$ is the consumption tax ratio. Taxes on investment goods are included only in the Carey and Rabesona (2004) tax ratio.

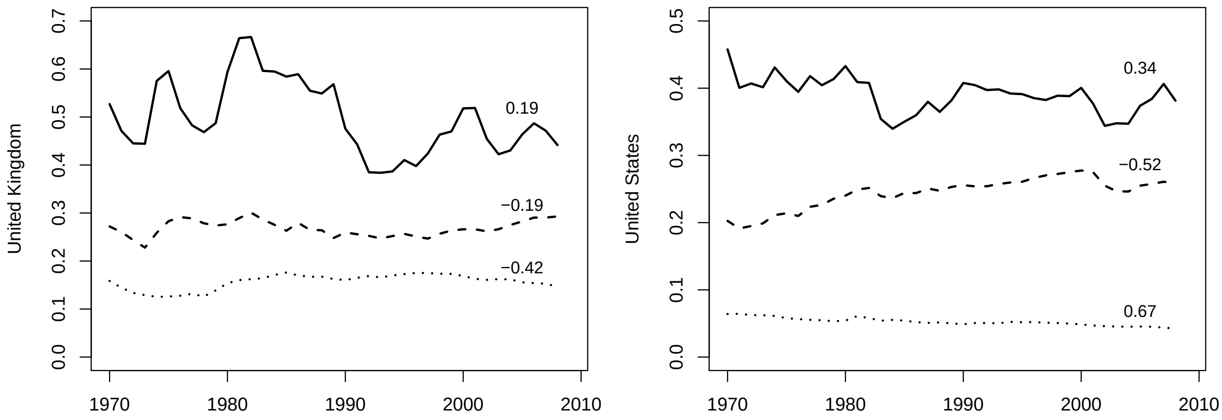
Volatility. Clearly, the variance can be measured in many ways. We employ two empirical approaches of either collapsing several periods into one observation by using non-overlapping (fixed) windows *or* by using rolling windows. Both concept are useful for the purpose of illustration and robustness checks. While the first approach discards information, the latter gives more data points but has more complicated statistical properties. We avoid those limitations later when treating the variance as a latent variable. For the first approach we use five-year windows gathering the variance of growth rates starting in 1970 until 2009. In the latter approach we use the five-year rolling variance of growth rates similar to Blanchard and Simon (2001). Both measures indicate that output volatility differs substantially over time and across countries (cf. Figure 1).

Taxes. We measure the average tax burden of a representative household at the macro level following Mendoza et al. (1994).[20] Accordingly, we use four different types of tax ratios: $LABOR$ measures the tax induced cost of dependent labor, i.e., taxes on labor income, security charges and payroll taxes; $CAPITAL$ measures the cost of capital through taxation, i.e., taxes on capital income, on the capital stock as well as on capital transactions; $CORP$ measures the income tax of corporations; $CONS$ measures the tax burden based on consumption expenditures, i.e., taxes on goods and services and excise taxes. Most notably, $CAPITAL$ contains taxes on property, including recurrent taxes on immovable property as well as taxes on financial and capital transactions. It includes the inheritance tax which, given our infinite horizon framework, is interpreted as a tax on wealth rather than a tax on income. To this end, our empirical average tax ratios are related to our theoretical marginal tax rates as summarized in Table 3. Accordingly, $LABOR$ and $CORP$ are taxes on income, whereas $CAPITAL$ comprises all the tax burden associated with capital income, capital flows and the stock of capital.

Figure 2 shows the time paths of tax ratios and their correlation with output volatility for the UK and the US. We find that changes in output volatility in the 1980s and the

[20]We also used modifications of Carey and Rabesona (2004) for effective tax rates with similar results.

Figure 2: Tax ratios and simple correlation with *observed* volatility



Notes: These figures show the time paths of tax ratios for capital ($CAPITAL$, solid), labor income ($LABOR$, dashed), consumption ($CONS$, dotted), and their correlation coefficient with the variance of annual growth rates of real GDP per capita (in percent) using fixed-windows (cf. Figure 1) for the UK (left panel) and the US (right panel).

1990s roughly coincide with abrupt changes of $CAPITAL$ following major tax reforms (cf. Figure 1). For example, the UK capital transfer tax (the inheritance tax) was cut from 75% in the early 1980s to 40% in the late 1980s and accompanied by an increase of the tax-base threshold from $25,000$£ in 1980 to $200,000$£ in 1995.[21] The data shows a substantial decline of the contribution of property tax revenues to $CAPITAL$. At the same time the UK was experiencing a period of low volatility in the late 1980s. In fact, we observe a positive correlation of $CAPITAL$ with output volatility for both the UK and the US. In Section 4 we confirm this anecdotal evidence and find similar empirical patterns between volatility and taxes in a panel of OECD countries even when controlling for other variables and/or time- and country-specific effects.

Other controls. As in most studies on volatility research, the control variables follow the construction of the measure of volatility, i.e., either by some fixed-window or rolling window approach.[22] Below we employ the following variables: $GROW$ denotes the mean of annual growth rates of real GDP per capita; $INFL$ and $INFLSD$ are the mean and standard deviation of the inflation rate; $GGDP$ and $GGDPSD$ denote the mean and standard deviation of government consumption as a percentage of GDP; $OPEN$ is the degree of openness measured by the ratio of exports plus imports to output; $XRSD$ is the standard deviation of the real effective exchange rate; $PRIVY$ measures financial development by the allocation of credit to the private sector as a percentage of GDP.

Alternatively we use the size of innovations to country-specific forecasting equations.

[21] Source: Institute for Fiscal Studies, http://www.ifs.org.uk/ff/indextax.php

[22] There is now a large literature on volatility estimation including Blanchard and Simon (2001), Denizer et al. (2002), Lensink and Scholtens (2004), Cecchetti et al. (2006), and Jaimovich and Siu (2009).

We include a constant, linear and quadratic time trends, and controls in forecasting equations for the inflation rate, government expenditures, and the real effective exchange rate. In particular, our variables are as follows: $INFLFI$ denotes the absolute value of innovations to the inflation forecast from a Phillips curve based measures of aggregate activity (two lags of HP-filtered real GDP per capita and two lags of the inflation rate in addition to the deterministic trends); $DGFI$ denotes the absolute value of innovations to the forecast of government-spending growth based on two lags of log GDP per capita and log government spending per capita; $XRFI$ denotes the absolute value of forecast innovations to the real effective exchange rate based on two autoregressive lags.

3.2 Estimation strategy

We are now prepared to address our empirical question: conditional on other controls, does output volatility vary systematically with tax rates? To this end, we consider the two related empirical frameworks. The first approach is regression-based using observed measures of volatility. The second approach considers volatility as a latent variable.

3.2.1 Observed volatility

For a quick look at the data, we begin estimating the following econometric model,

$$\log(\sigma_{it}^2) = \alpha_i + \lambda_t + \beta' x_{it} + \gamma' z_{it} + u_{it}, \tag{20}$$

in which we use *observed* volatility measures. σ_{it}^2 denotes the variance of annual real growth rates of real GDP per capita at time $t = 1, ..., T$ with either $T = 8$ (fixed-window) or $T = 35$ (rolling-window) observations for each country $i = 1, ..., N$.[23] Following our strategy in (18), $\log(\sigma_{it}^2)$ is a linear function of country and time fixed effects, α_i and λ_t, taxes, x_{it}, other controls, z_{it}, and an residual term, u_{it}. By using a log-linear specification in (20) we ensure that the fitted values for σ_{it}^2 are strictly positive and we can directly interpret the estimated parameters as semi-elasticities.

Our specification in (20) with fixed-windows emphasizes the cross-sectional dimension relative to the time dimension, and thus suitable for the study of asymptotic properties in the data. This approach therefore is complementary to both the rolling-window and the more general likelihood-based approach below. We obtain parameter estimates using the least square dummy variable (LSDV) methodology. To avoid that results are driven by few outliers, we also use iterated weighted least squares (IWLS) estimation.[24]

[23] See Blanchard and Simon (2001) and Jaimovich and Siu (2009) for similar specifications. One concern in this specification is that the results might be spurious. As the time horizon using the fixed-window specification is relatively short, this is not as problematic as using rolling-windows.

[24] It is well known that the least squares estimator is particularly sensitive to small numbers of atypical data points when the sample size is small or moderate. Using regression diagnostics for influential data points (leave-one-out deletion) suggests that a small number of observations have potentially large effects.

3.2.2 Unobserved volatility

Our second approach allows us to estimate business cycle volatility. The methodology goes back to Ramey and Ramey (1995), who used a similar approach to analyze the effect of government spending-induced volatility on growth, and was recently used in Jaimovich and Siu (2009), who analyze the effect of demographics on output volatility.

For our purposes, we use (19) and the following empirical framework linking taxes to the conditional variance which here is equivalent to output growth variance,

$$\Delta y_{it} = \theta_i + \epsilon_{it}, \quad \text{where} \quad \epsilon_{it} \sim \mathcal{N}(0, \sigma_{it}^2), \tag{21a}$$

$$\log(\sigma_{it}^2) = \alpha_i + \lambda_t + \beta' x_{it} + \gamma' z_{it}. \tag{21b}$$

Δy_{it} is the annual growth rate of real GDP per capita for country $i = 1, ..., N$ at time $t = 1, ..., T$, expressed in log differences, θ_i is a country-specific mean and σ_{it}^2 denotes the variance of the residuals in the growth equation, ϵ_{it}. Our primary focus is on the *unobserved* volatility process (21b) which models $\log(\sigma_{it}^2)$ as a linear function of country and time fixed effects, α_i and λ_t, tax rates, x_{it}, and other controls, z_{it}.

There seems to be consensus among economists that volatility changes over time. In addition to country-specific controls α_i, we allow for time-specific breaks in the conditional variance based on Stock and Watson (2005) and other recent events occurring broadly across countries to account for structural breaks in volatility. In particular we include $D = 6$ time dummies λ_t for the period until 1980 (breaks in the UK 1979:4-82:1 and Italy 1979:3-82:4), for 1980-84 (break in the US 1982:4-85:3), for 1985-90 (break in Canada 1990:4-93:1), for 1991-1995 (break in Germany 1992:3-95:2), for 1996-2000 (until the new economy bubble burst), and for 2001-2007 (until the recent financial crisis in 2008).

Our benchmark specification (21) is nested in the class of autoregressive conditional heteroscedasticity (ARCH) models. We estimate the parameter vector $\omega \equiv (\theta, \alpha, \lambda, \beta, \gamma)$ jointly with the conditional variance σ_{it}^2 using the maximum likelihood (ML) approach. The log-likelihood function reads (cf. Engle 1982, Bollerslev 1986)

$$\ell(\omega)_{NT} = -\tfrac{1}{2} \sum_{t=1}^{T} \sum_{i=1}^{N} \log(\sigma_{it}^2) - \tfrac{1}{2} \sum_{t=1}^{T} \sum_{i=1}^{N} \epsilon_{it}^2 / \sigma_{it}^2. \tag{22}$$

Under sufficient regularity conditions, the ML estimator is consistent and asymptotic normal. We obtain asymptotic standard errors using the outer product estimate.

4 Empirical results

This section holds the estimation results. Following our estimation strategy, we use observed volatility measures to get a general idea about effects present in the data. We then fully exploit the panel structure by treating unobserved variances as parameters.

Table 4: Static panel estimation, *observed* volatility measures (five-year windows)

OECD		LSDV (fixed-window)	LSDV (fixed-window)	LSDV (rolling-window)	LSDV (rolling-window)
$LABOR_{it}$	β_1	−6.16 (2.82) *	−8.29 (2.80) **	−6.54 (1.59) ***	−6.36 (1.61) ***
$CAPITAL_{it}$	β_2	7.89 (2.84) **	5.70 (2.44) *	6.32 (1.39) ***	3.36 (1.42) *
$CONS_{it}$	β_3	7.04 (3.67) ·	6.89 (3.34) *	6.57 (2.99) **	9.91 (2.08) ***
$CORP_{it}$	β_4	−3.85 (2.50)	−2.64 (2.30)	−4.72 (1.14) ***	−3.24 (1.18) **
$GROW_{it}$	γ_1		−19.10 (6.94) **		−9.83 (1.74) ***
$PRIVY_{it}$	γ_2		0.24 (0.32)		−0.14 (0.19)
$INFL_{it}$	γ_3		−11.47 (3.29) ***		0.85 (1.24)
$INFLSD_{it}$	γ_4		26.49 (5.51) ***		9.82 (3.51) **
$GGDP_{it}$	γ_5		−2.39 (6.05)		4.49 (3.53)
$GGDPSD_{it}$	γ_6		34.70 (19.28) ·		31.21 (11.68) **
$XRSD_{it}$	γ_7		0.11 (3.57)		4.74 (2.00) *
$OPEN_{it}$	γ_8		1.33 (1.04)		0.63 (0.50)
Degrees of freedom		129	120	669	624
Adjusted R-squared		0.30	0.40	0.34	0.38
F-statistic		3.23	3.81	13.42	11.95
Country fixed effects	α_i	yes	yes	yes	yes
Time fixed effects	λ_t	yes	yes	yes	yes

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '·' 0.1 ' ' 1

Notes: This table reports the semi-elasticities of the fixed-effects model (20) using the least square dummy variable approach (LSDV) explaining the variance of annual growth rates of real GDP per capita. Standard errors of White's heteroscedasticity-consistent covariance matrix estimators (HCCME) are in parentheses. We use one-period lagged controls for the rolling window approach.

4.1 Volatility and taxes

4.1.1 Observed volatility

A quick answer to our empirical question is in Table 4. It presents our estimates for the semi-elasticities of various controls on *observed* output growth volatility (percentage change of σ_{it}^2 given a percentage point increase of the control) for (20). We consider this specification as a simple and informative way to illustrate the effects of taxes on output volatility. In the next subsection we discuss the robustness of our results to more elaborate specifications (with alternative specification of the controls). Output growth variance can be explained by various fundamentals (between 30-40 percent of its variability). Our key parameter vector of interest is β, which links the empirical tax ratios to volatility.

Table 4 provides strong empirical evidence for tax effects on output volatility in line with our theoretical priors (compare with Tables 2 and 3). We roughly get a similar picture for the fixed-window and the rolling-window approaches. To summarize, effects of taxes on corporate income ($CORP$) and on labor income ($LABOR$) are statistically significant and negative (columns 3 and 4). Holding constant other variables an increase of $LABOR$ by 1 percentage point decreases output volatility by about 6.2% to 8.3%.

Both the capital tax ($CAPITAL$) and the consumption tax ($CONS$) are positively related. While the first result can be explained by the effects of the tax on wealth (which dominates the income effect), the effect of the consumption tax is more surprising but turns out not being robust across other specifications below.

Other control variables are in line with the literature. We find a negative link between mean and the variance of output growth rates: countries growing 1 percentage points below average ($GROW$) are experiencing 19.1% (or 9.8%) higher output growth variance using fixed-windows (rolling-windows). Our measures of variability of the effective exchange rate ($XRSD$), the government consumption to GDP ratio ($GGDPSD$) and the inflation rate ($INFLSD$) are positively related. Somewhat controversial, we find a negative effect of the mean inflation rate ($INFL$). In related work Denizer et al. (2002) find no statistically significant effect, a positive link is documented in Lensink and Scholtens (2004). One explanation is a potential endogeneity problem arising with contemporaneous variables in the fixed-window approach (which we address below). The effects for the government consumption to GDP ratio ($GGDP$), openness ($OPEN$), and financial development ($PRIVY$) are not significantly different from zero.

Figure A.1 in Appendix A.5 plots the pooled observations for taxes against output volatility (fixed-windows). Figure A.2 shows the partial correlation of tax ratios with observed output growth variance (Table 4, column 2), i.e., the tax effects after controlling for other variables and fixed-effects. In the appendix we also provide results from a robust IWLS procedure with bootstrapped standard errors (cf. Table A.1). As a result, the overall pattern for the effects on taxes on output growth volatility does not change. However, the estimates generally loose precision for the fixed-windows approach, while effects are even more pronounced in the rolling-window approach.

4.1.2 Unobserved volatility

Complementary to our results based on observed volatility (fixed-windows) we examine the relationship between output volatility and taxes by efficiently exploiting the time dimension of our panel data set. At the same time, the following approach allows us to avoid issues related to serial correlation of residuals from the use of rolling windows in measuring volatility. We estimate the system (21) for similar specifications as in Table 4, replacing control variables based on standard deviations by innovations to country-specific forecasting equations.[25] To avoid endogeneity problems arising with contemporaneous variables we use one-period lagged values for our explanatory variables.

Table 5 presents the ML estimates based on (22). Comparing the likelihood-based to

[25] We exclude $OPEN$ which turned out to be insignificant and uninformative for the whole estimation approach, while its effect seems to be fully captured by $XRFI$.

Table 5: Static panel estimation, treating variances as parameters

OECD		MLE (taxes only)	MLE (benchmark)	MLE (no time effects)	MLE (no time effects)
$LABOR_{i,t-1}$	β_1	−8.49 (2.87) **	−5.46 (3.19) ·	−14.72 (2.22) ***	−9.97 (3.00) ***
$CAPITAL_{i,t-1}$	β_2	8.11 (2.46) **	8.20 (2.66) **	9.84 (2.24) ***	8.89 (2.48) ***
$CONS_{i,t-1}$	β_3	4.23 (4.25)	7.15 (4.34)	−0.56 (3.52)	−3.97 (4.06)
$CORP_{i,t-1}$	β_4	−2.42 (1.98)	−2.35 (2.11)	−3.81 (1.68) *	−2.87 (1.84)
$GROW_{i,t-1}$	γ_1		−9.51 (0.43) *		−9.86 (3.85) *
$PRIVY_{i,t-1}$	γ_2		0.04 (0.43)		−0.27 (0.27)
$INFL_{i,t-1}$	γ_3		−0.22 (2.62)		2.59 (2.41)
$INFLFI_{i,t-1}$	γ_4		9.00 (5.01) ·		10.77 (4.86) *
$GGDP_{i,t-1}$	γ_5		−18.32 (7.30) *		−9.87 (6.68)
$DGFI_{i,t-1}$	γ_6		1.34 (4.04)		1.03 (4.10)
$XRFI_{i,t-1}$	γ_7		0.27 (2.54)		0.72 (2.31)
Degrees of freedom		642	635	648	641
Log-likelihood		−1804.1	−1813.0	−1787.9	−1799.8
Country fixed effects	θ_i	yes	yes	yes	yes
Country fixed effects	α_i	yes	yes	yes	yes
Time fixed effects	λ_t	yes	yes	no	no

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '·' 0.1 ' ' 1

Notes: This table reports the semi-elasticities of the joint estimation of (21) using maximum likelihood, explaining the conditional variance of annual growth rates of real GDP per capita. Asymptotic standard errors are in parentheses.

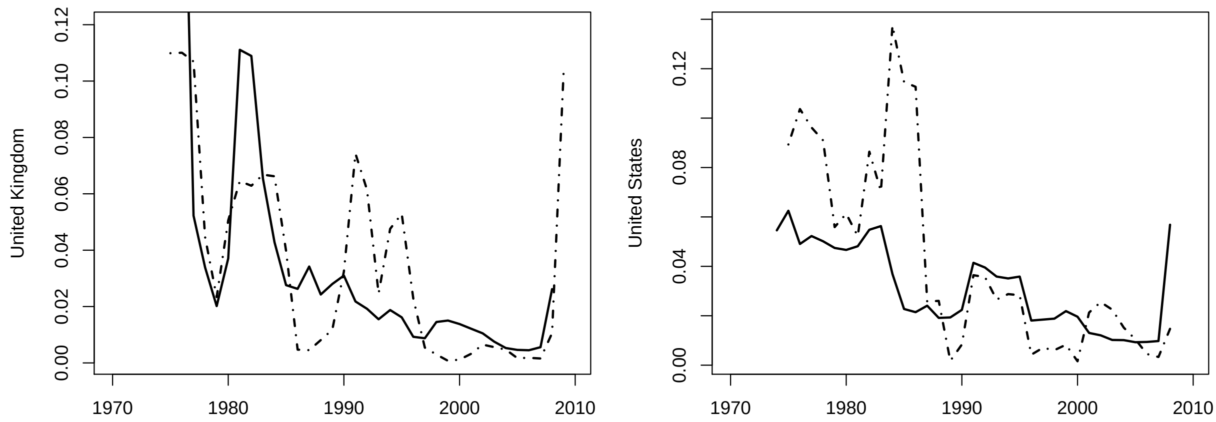
the regression-based estimates in Table 4 gives similar results: we find that $LABOR$ and $CORP$ are negatively related to output growth volatility, while $CAPITAL$ is associated with higher volatility. Moreover, the order of magnitude for our tax ratios is roughly comparable with the initial estimates. For example, an increase in $CAPITAL$ by 1 percentage point increases the conditional variance by 8.1% in column 1.[26] We interpret this finding as a strong empirical evidence for a link between taxes and output growth volatility ($CONS$ is no longer statistically significant).

It is notable that the empirical tax effects are substantially higher than implied by the theory (compare with semi-elasticities of Table 2). In our modeling framework without rigidities and with constant labor supply this result should not come as a surprise. Given our theoretical results we also expect tax effects on labor supply and its variability. We believe that a more elaborate framework is needed to quantitatively match our empirical findings. Since recent results trace some of the decline in US output volatility to less volatile labor input, an extension in this direction seems promising (cf. Stiroh 2009).

Other controls that significantly contribute to output volatility in our benchmark specification (Table 5, column 2) are the growth rates of real GDP per capita ($GROW$),

[26] Bilbiie et al. (2008) stress the importance of 'asset market participation'. Since $CAPITAL$ includes taxes on financial and capital transactions, this could induce the negative association with volatility: a lower tax may increase participation in asset markets and lowers output growth volatility.

Figure 3: Static panel, observed vs. estimated volatility (benchmark model)



Notes: These figures plot estimated conditional variance (solid) and observed five-year rolling variance (dot-dashed) of annual growth rates of real GDP per capita (in percent) for the UK (left panel) and the US (right panel) starting in 1970 (cf. Table 5, column 2).

innovations to the inflation forecast ($INFLFI$), and the government consumption to GDP ratio ($GGDP$). For illustration, controlling for time effects we find evidence for an anti-cyclical fiscal policy.[27] Increasing $GGDP$ by 1 percentage point slightly decreases output growth volatility by 0.18%. This estimate indicates that output growth volatility empirically is much more sensitive to changes in tax rates (measured in percent) than changes in the government consumption to GDP ratio. We do not find statistically significant effects for financial development ($PRIVY$), the inflation rate ($INFL$), and innovations to government-spending growth ($DGFI$) and to the real effective exchange rate ($XRFI$). The inclusion of time effects certainly improves the fit but does not change the overall pattern (compare columns 2 and 4).

Figure 3 illustrates the estimated volatility patterns for the UK and the US for our benchmark specification (Table 5, column 2). Overall, the time paths and cross-country patterns are well captured by our benchmark model.

4.2 Further robustness results

This section provides further evidence on the link between output volatility and taxes. It turns out that our benchmark results are robust to extensions of our empirical approach when accounting, e.g., for (local) non-stationarity of the conditional variance and taxes (Section 4.2.1), transitional dynamics in the growth equation (Section 4.2.2) and/or including a feedback from volatility to the growth equation (Section 4.2.3).

[27] By including time effects we are removing breaks in volatility which occur broadly across countries.

4.2.1 Dynamic panel estimation

One concern is that the observed correlation might capture trends in both output growth variance and tax rates. For example, the effect of $LABOR$ substantially increases when excluding time-specific effects λ_t (cf. Table 5, columns 1 and 3), which indicates that $LABOR$ might be picking up some (local) non-stationarity. In our specification (21) we treat our variables as $I(0)$. If taxes and output volatility are $I(1)$, our results may be either spurious or superconsistent. The latter is true for a cointegrating relationship. Obviously, a formal test of cointegration cannot be applied to unobserved variables. Since our focus is on a long-run relationship between taxes and output volatility, we extend our analysis (21) to a *dynamic* approach, where log output growth variance and taxes are assumed to be integrated of order $I(1)$. Hence, we estimate the system

$$\Delta y_{it} = \theta_i + \epsilon_{it}, \quad \text{where} \quad \epsilon_{it} \sim \mathcal{N}(0, \sigma_{it}^2), \tag{23a}$$

$$\Delta \log(\sigma_{it}^2) = \alpha_i + \lambda_t + \vartheta' \Delta x_{it} + (\rho - 1)(\log(\sigma_{i,t-1}^2) - \beta' x_{i,t-1}) + \gamma' z_{it}. \tag{23b}$$

Our empirical specification (23) closely follows the cointegration idea: suppose taxes and output volatility are *not* cointegrated. In order to balance the time series property that the left-hand side of (23b) is stationary, the parameter ρ cannot be different from 1 to obtain stationarity on the right-hand side.[28] Observe that the system (23) is nested in an generalized autoregressive conditional heteroscedasticity (GARCH) model.

To make our point for a cointegrating relationship more explicit we reconsider the case of *observed* volatility (fixed-window) and the following error correction model,

$$\Delta \log(\sigma_{it}^2) = \alpha_i + \lambda_t + \vartheta' \Delta x_{i,t} + (\rho - 1)(\log(\sigma_{i,t-1}^2) - \beta' x_{i,t-1}) + \gamma' z_{it} + v_{it}. \tag{24}$$

The equation (24) explains the changes in $\log(\sigma_{it}^2)$ by country and time fixed-effects, α_i and λ_t, changes of tax rates x_{it} around their long-run trend, plus a part $(\rho - 1)$ of the error correction $\log(\sigma_{i,t-1}^2) - \beta' x_{i,t-1}$ (the equilibrium error in the model of cointegration), stationary controls z_{it}, and an uncorrelated error term v_{it} with mean zero and equal variance. We interpret values $\rho < 1$ as follows. If volatility is lower than the equilibrium value, then volatility tends to increase in the next period (and vice versa).

Table 6 presents estimation results for the system (23).[29] Our estimated parameters are confirmative of a long-run relationship between taxes and output volatility: the null hypothesis of $\rho = 1$ (no cointegration) would be rejected at any conventional significance level. The equilibrium error indeed is informative for future changes in output growth

[28] We are not aware of research on cointegration within the conditional variance equation. According to the cointegration principle, however, one should add an error term to equation (23b) as in (24). This would lead to a stochastic volatility model which is an interesting path for future research.

[29] To start with, we need pre-sample estimates for σ_{it}^2 for $t \leq 0$. As a natural choice, we follow Bollerslev (1986) and use country-specific sample analogues $\sigma_{i,0}^2 = T^{-1} \sum_t \epsilon_{it}^2$.

Table 6: Dynamic panel estimation, treating variances as parameters

OECD		MLE (taxes only)	MLE (full model)	MLE (no time effects)	MLE (no time effects)
$\Delta LABOR_{i,t}$	ϑ_1	−7.99 (6.66)	−3.77 (6.99)	−1.41 (6.00)	−2.18 (6.81)
$\Delta CAPITAL_{i,t}$	ϑ_2	4.06 (5.04)	3.96 (5.34)	5.18 (3.73)	3.60 (4.86)
$\Delta CONS_{i,t}$	ϑ_3	−10.44 (9.08)	−10.66 (9.62)	−11.96 (8.16)	−12.49 (8.75)
$\Delta CORP_{i,t}$	ϑ_4	−1.19 (3.57)	−0.98 (3.79)	−1.90 (2.85)	−1.19 (3.45)
$LABOR_{i,t-1}$	β_1	−7.75 (2.76) **	−5.63 (3.30) ·	−15.74 (2.70) ***	−10.04 (3.43) **
$CAPITAL_{i,t-1}$	β_2	6.76 (2.68) *	7.43 (3.04) *	10.96 (2.68) ***	8.96 (3.07) **
$CONS_{i,t-1}$	β_3	1.59 (4.21)	5.04 (4.84)	−0.39 (3.59)	3.32 (4.35)
$CORP_{i,t-1}$	β_4	−1.67 (2.17)	−1.77 (2.45)	−4.53 (2.06) *	−2.92 (2.38)
$GROW_{i,t-1}$	γ_1		−11.58 (4.77) *		−9.62 (4.31) *
$PRIVY_{i,t-1}$	γ_2		0.07 (0.43)		−0.31 (0.28)
$INFL_{i,t-1}$	γ_3		−1.24 (2.88)		1.73 (2.42)
$INFLFI_{i,t-1}$	γ_4		10.10 (5.25) ·		12.31 (5.09) *
$GGDP_{i,t-1}$	γ_5		−17.63 (8.03) *		−9.66 (6.38)
$DGFI_{i,t-1}$	γ_6		0.94 (4.32)		0.02 (4.27)
$XRFI_{i,t-1}$	γ_7		−0.37 (2.55)		0.57 (2.48)
$\sigma_{i,t-1}$	ρ	−0.39 (0.29)	−0.03 (0.23)	0.43 (0.30)	0.14 (0.21)
Country fixed effects	θ_i	yes	yes	yes	yes
Country fixed effects	α_i	yes	yes	yes	yes
Time fixed effects	λ_t	yes	yes	no	no
Degrees of freedom		630	623	636	629
Log-likelihood		−1789.8	−1799.2	−1776.0	−1787.2

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '·' 0.1 ' ' 1

Notes: This table reports the semi-elasticities of the joint estimation of (21) using maximum likelihood, explaining the conditional variance of annual growth rates of real GDP per capita. Asymptotic standard errors are in parentheses.

volatility. Our results indicate an equilibrium relationship between both $LABOR$ and $CAPITAL$ with output growth volatility. Similar to our benchmark model $CORP$ is not robust across our specifications, while $CONS$ is not informative.

Comparing the results to our benchmark estimates (Tables 5, column 2), we find similar quantitative effects. An increase in $CAPITAL$ by 1 percentage point would lead to an increase of the equilibrium conditional variance by 7.4% (Table 6, column 2).[30] Suppose this tax change would turn the equilibrium error negative since current volatility is lower than its new equilibrium value. This disequilibrium causes a tendency to positive changes of future output volatility, $\Delta \log(\sigma_{it})$. Overall, our results strengthen the idea of a long-run equilibrium relationship between taxes and output volatility.

[30]Table A.2 in Appendix A.5 holds the results estimating (24) for the fixed-window approach. We find evidence for a cointegrating relationship: our estimate of ρ is between −0.21 and −0.12. Using the bias-correction of Bun and Carree (2005) for dynamic panels does not change our results.

4.2.2 Transitional dynamics in the growth equation

Up to this point our results show a long-run equilibrium relationship between taxes and output growth volatility. Our benchmark specification (21), based on an asymptotic result, is motivated by our theoretical model where the second moment rather than the first moment is affected by taxes. In this section we extend our empirical analysis to include transitional dynamics in the growth equation (21a).

In particular, we are interested in this approach for the following reason. Although there is mixed empirical evidence on tax effects on output growth, one serious concern is that taxes could effect output growth rates in the short run. Our benchmark model simply captures such transitional effects by the residual term. It is therefore important to study whether the observed correlation in the conditional variance equation is due to short-term effects of taxes in the growth equation.[31]

We extend our empirical specification (21) as follows

$$\Delta y_{it} = \theta_i + \delta' x_{it} + \varphi_1 \Delta y_{i,t-1} + \varphi_2 \Delta y_{i,t-2} + \epsilon_{it}, \quad \epsilon_{it} \sim \mathcal{N}(0, \sigma_{it}^2), \tag{25a}$$

$$\log(\sigma_{it}^2) = \alpha_i + \lambda_t + \beta' x_{it} + \gamma' z_{it}. \tag{25b}$$

This model now allows for possible tax effects in the growth equation and for greater flexibility in fitting the time-series property of the data.

Table A.3 in the appendix presents the results from this procedure. Our results show that by including the endogenous variables with lags substantially helps to explain the variation of growth rates. In addition, we find some empirical evidence of a negative effect of $CAPITAL$ on output growth. The bottom line, however, is that our key parameter β, which links the tax ratios to output growth volatility, generally is not affected by the inclusion of lagged endogenous variables and/or taxes in the growth equation (25a).

4.2.3 The link between volatility and growth

This section allows for a possible feedback effect in the growth equation following Ramey and Ramey (1995) to analyze the link between volatility and growth. This is important since tax effects on growth simply could emerge through the negative effects of output volatility on growth. Their framework is nested in a generalized autoregressive conditional heteroscedasticity-in-mean (GARCH-M) model. We add further controls including taxes in the conditional variance equation and confirm a negative relationship.[32]

[31] Our country-specific fixed effects capture the Levine-Renelt growth regression variables. We include lagged variables of the growth rate to control for transitional dynamics in the growth equation.

[32] In general the GARCH-M model has a drawback as no sufficient conditions for consistency and asymptotic normality are yet known. Following common practice, we assume that the maximum likelihood estimator is consistent and asymptotic normal (see Nelson 1991).

Table 7: New evidence on the link between volatility and growth

OECD		MLE (taxes only)	MLE (full model)	MLE (extended)	MLE (baseline)
$LABOR_{i,t-1}$	δ_1	-0.25 (0.08) **	-0.18 (0.07) **		
$CAPITAL_{i,t-1}$	δ_2	0.11 (0.06) ·	0.07 (0.06)		
$CONS_{i,t-1}$	δ_3	-0.03 (0.08)	-0.01 (0.07)		
$CORP_{i,t-1}$	δ_4	-0.07 (0.05)	-0.04 (0.04)		
$GROW_{i,t-1}$	ϑ_1	0.28 (0.04) ***	0.18 (0.07) *	0.33 (0.04) ***	0.35 (0.04) ***
$GROW_{i,t-2}$	ϑ_2	-0.12 (0.04) **	-0.07 (0.04) *	-0.08 (0.04) *	-0.10 (0.04) **
$LABOR_{i,t-1}$	β_1	-13.23 (2.09) ***	-10.56 (2.33) ***	-8.46 (2.48) ***	
$CAPITAL_{i,t-1}$	β_2	9.37 (2.14) ***	8.89 (2.28) ***	8.91 (2.08) ***	
$CONS_{i,t-1}$	β_3	1.51 (3.17)	3.71 (3.32)	6.37 (3.06) *	
$CORP_{i,t-1}$	β_4	-5.83 (1.75) ***	-4.74 (1.85) *	-5.14 (1.60) **	
$GROW_{i,t-1}$	γ_1		-5.37 (3.30)		
$PRIVY_{i,t-1}$	γ_2		-0.09 (0.18)		
$INFL_{i,t-1}$	γ_3		3.81 (1.41) **		
$INFLFI_{i,t-1}$	γ_4		4.55 (2.57) ·		
$GGDP_{i,t-1}$	γ_5		-8.13 (3.61) *		
$DGFI_{i,t-1}$	γ_6		0.39 (1.97)	4.05 (3.10)	5.34 (3.08) ·
$XRFI_{i,t-1}$	γ_7		0.41 (1.12)		
$\sigma_{i,t}$	ν	-2.66 (0.68) ***	-2.32 (0.56) ***	-0.98 (0.17) ***	-0.91 (0.17) ***
Degrees of freedom		635	628	638	642
Log-likelihood		-1875.6	-1893.5	-1866.3	-1849.0
Country fixed effects	θ_i	yes	yes	yes	yes
Country fixed effects	α_i	yes	yes	yes	yes
Time fixed effects	λ_t	yes	yes	yes	yes

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '·' 0.1 ' ' 1

Notes: This table reports the semi-elasticities of the joint estimation of (26) using maximum likelihood, explaining the conditional variance of annual growth rates of real GDP per capita. Asymptotic standard errors are in parentheses.

To this end we finally generalize (21) and (25) as follows

$$\Delta y_{it} = \theta_i + \delta' x_{it} + \varphi_1 \Delta y_{i,t-1} + \varphi_2 \Delta y_{i,t-2} + \nu\sigma_{it} + \epsilon_{it}, \quad \epsilon_{it} \sim \mathcal{N}(0, \sigma^2_{it}), \tag{26a}$$

$$\log(\sigma^2_{it}) = \alpha_i + \lambda_t + \beta' x_{it} + \gamma' z_{it}. \tag{26b}$$

Observe that compared to system (25) only the conditional variance now appears as an additional control in the growth equation.

Table 7 presents the results of this procedure. To relate our findings to Ramey and Ramey (1995, Table 4), we estimate (26) using government-spending induced volatility ($DGFI$) in the conditional variance equation and two lags of the endogenous variable the growth equation, and with fixed-effects for both specifications (column 4). Then, we compare our results to cases where we add taxes and other controls to the conditional variance equation and/or to the growth equation (columns 1 to 3).

Our estimates confirm both a robust link between output volatility and taxes and a negative partial correlation between output volatility and growth. As in our benchmark specification, $LABOR$ and $CORP$ decrease output growth volatility, $CAPITAL$ has a positive correlation and the effect of $CONS$ is not robust across specifications.

We also find that $DGFI$ is informative in the Ramey-Ramey specification. When we include further controls in the variance equation, however, $DGFI$ turns insignificant (Table 7, columns 2 and 4). In contrast, the link between volatility and growth becomes more pronounced . Since $GROW$ does no longer significantly contribute to the variation in the conditional variance (column 2), the negative correlation in the benchmark model (Table 5, column 2) may simply reflect the missing feedback effect in the growth equation. Nevertheless, this cannot be interpreted as a causal finding.

Other controls which significantly contribute to the conditional variance of output growth rates are the inflation rate ($INFL$), the innovations to the inflation forecast ($INFLFI$) and the government consumption to GDP ratio ($GGDP$). Observe that now all controls (including $INFL$) appear in line with the literature: a higher inflation rate, higher innovations to the inflation forecast, and a lower GDP share of government consumption are associated with higher output volatility (Table 7, column 2).

Though our results also show a negative correlation between $LABOR$ with output growth rates, the overall effect on output growth is not obvious. For example, a higher $LABOR$ directly decreases output growth rates but indirectly increases output growth rates by lowering its variance. This finding could explain why previous studies find only mixed empirical evidence of tax effects on output growth. Changes in taxes also trigger changes in output volatility, which in turn could affect output growth rates.

5 Conclusions

The aim of this paper is to shed light on the link between taxes and output volatility. We start from a theoretical perspective and demonstrate how distortional taxes affect the variability of macroaggregates in the stochastic neoclassical model. Contrary to conventional perceptions, we show that the second moment rather than the mean of output growth rates is affected by taxes. Based on a closed-form solution we illustrate that consumers' decisions have effects on output volatility because they affect the variability of capital rewards through their consumption-savings decision. We then calibrate the model in order to obtain the tax semi-elasticities on output growth variance.

Taking the model to the data, we make use of the heterogeneity of tax rates to estimate tax effects on output growth variance using panel estimation techniques. Our study brings out some strong empirical regularities in output volatility among OECD countries. For several measures of volatility and estimation techniques we find convincing evidence that taxes are key determinants, robust, and economically important in explaining differences across countries and over time. Accounting for potential non-stationarity of taxes and output volatility, we find empirical evidence for a cointegrating relationship.

In accordance with our theory, we find that tax effects are not unidirectional: while the labor and the corporate income tax ratios are negatively correlated, the capital tax ratio is positively correlated with output volatility, the consumption tax ratio has no significant effect. We also confirm a strong empirical link between volatility and growth (Ramey and Ramey 1995). Allowing for more heterogeneity in the conditional variance equation indeed strengthens the observed empirical link.

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A Appendix

A.1 The model

A.1.1 The household's budget constraint (6)

Let nominal wealth be $(1 + \tau_c)p_t^C a_t = k_t v_t$ where k_t is the individual's capital, v_t is the value of one unit of capital, and p_t^C is the producer price of the consumption good. For positive investment the price of an installed good equals the price of a new investment good, $v_t = (1+\tau_k)p_t^K$, hence

$$a_t = \frac{1+\tau_k}{1+\tau_c}k_t. \tag{27}$$

Households receive net capital income $(1-\tau_i)p_t^Y r_t k_t$ and net labor income $(1-\tau_i)p_t^Y w_t$ (after-tax value marginal products), which is used for saving and consumption. Define savings $p_t^Y s_t \equiv (1-\tau_i)p_t^Y(r_t k_t + w_t) - (1+\tau_c)p_t^C c_t$. A fraction $\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a$ of the capital stock disappears as a result of depreciation and taxation,

$$dk_t = \left\{\frac{p_t^Y s_t}{(1+\tau_k)p_t^K} - \frac{1-\tau_i}{1+\tau_k}\delta k_t - \tau_a k_t\right\}dt. \tag{28}$$

The relationship in (28) shows that a positive tax on wealth, τ_a, simply increases the rate of effective depreciation. We show below τ_a also applies to real wealth, a_t, and not only to the number of machines, k_t. Using (27), the budget constraint reads

$$da_t = \frac{1+\tau_c}{1+\tau_k}\left\{\frac{s_t}{1+\tau_k} - \frac{1-\tau_i}{1+\tau_k}\delta k_t - \tau_a k_t\right\}dt. \tag{29}$$

Inserting s_t, replacing k_t with the definition in (27) and using (8) gives

$$\begin{aligned} da_t &= \frac{1+\tau_k}{1+\tau_c}\left\{\frac{1-\tau_i}{1+\tau_k}r_t k_t + \frac{1-\tau_i}{1+\tau_k}w_t - \frac{1+\tau_c}{1+\tau_k}c_t - \frac{1-\tau_i}{1+\tau_k}\delta k_t - \tau_a k_t\right\}dt, \\ &\equiv \left\{\left(\frac{1-\tau_i}{1+\tau_k}(r_t-\delta) - \tau_a\right)a_t + \frac{1-\tau_i}{1+\tau_c}w_t - c_t\right\}dt, \end{aligned}$$

where factor rewards are, $w_t = \frac{\partial Y_t}{\partial L} \equiv Y_L = (1-\alpha)Y_t/L$, and $r_t = \frac{\partial Y_t}{\partial K_t} \equiv Y_K = \alpha Y_t/K_t$.

A.1.2 The budget constraint of the government (4)

We start by summing up the budget constraint (6) using $\sum_{i=1}^{L} a_{t,i} = La_t$ to obtain

$$Lda_t = \left\{\left(\frac{1-\tau_i}{1+\tau_k}(r_t-\delta) - \tau_a\right)La_t + L\frac{1-\tau_i}{1+\tau_c}w_t - C_t\right\}dt,$$

where C_t denotes $C_t = Lc_t$. Transforming a_t into units of the capital stock from (27),

$$a_t = \frac{1+\tau_k}{1+\tau_c}Lk_t/L \equiv \frac{1+\tau_k}{1+\tau_c}K_t/L, \tag{30}$$

and insert it in the aggregated budget constraint yields

$$\begin{aligned} d\left(\frac{1+\tau_k}{1+\tau_c}K_t\right) &= \left\{\frac{1+\tau_k}{1+\tau_c}K_t\left(\frac{1-\tau_i}{1+\tau_k}(r_t-\delta) - \tau_a\right) + L\frac{1-\tau_i}{1+\tau_c}w_t - C_t\right\}dt, \\ \Leftrightarrow\ dK_t &= \left\{\frac{1-\tau_i}{1+\tau_k}(Y_K K_t + Y_L L) - \left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)K_t - \frac{1+\tau_c}{1+\tau_k}C_t\right\}dt, \\ &= \left\{\frac{1-\tau_i}{1+\tau_k}Y_t - \left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)K_t - \frac{1+\tau_c}{1+\tau_k}C_t\right\}dt, \end{aligned}$$

where we used Euler's theorem, that is $Y_t = Y_K K_t + Y_L L$ in the last step. Finally, we rewrite $dK_t/dt = I_t - \delta K_t$, multiply by $(1+\tau_k)$ and insert $(1+\tau_k)I_t$ from (7),

$$\begin{aligned}(1+\tau_k)(I_t - \delta K_t) &= (1-\tau_i)(Y_t - \delta K_t) - \tau_a(1+\tau_k)K_t - (1+\tau_c)C_t \\ \Leftrightarrow\ Y_t - C_t - I_t &= \tau_k(I_t - \delta K_t) + \tau_i(Y_t - \delta K_t) + \tau_a(1+\tau_k)K_t + \tau_c C_t \equiv G.\end{aligned}$$

A.1.3 The maximized Bellman equation

Define the value of an optimal program of (5) as

$$V(a_0, A_0) = \max_{\{c_t\}_{t=0}^{\infty}} \{U_0\},$$

which denotes the present discounted value of utility evaluated along the optimal program. Following the same steps as in Posch (2009), the Bellman equation reads

$$\begin{aligned}\rho V(a_0, A_0) &= \max_{c_0}\left\{u(c_0) + V_a(a_0, A_0)\left(\left(\frac{1-\tau_i}{1+\tau_k}(r_0 - \delta) - \tau_a\right)a_0 + \frac{1-\tau_i}{1+\tau_c}w_0 - c_0\right)\right. \\ &\qquad \left. + V_A A_t \mu + \tfrac{1}{2}V_{AA}A_t^2\eta^2\right\},\end{aligned}$$

The first-order condition reads

$$u'(c_0) = V_a(a_0, A_0), \tag{31}$$

making consumption a function of the state variables, $c = c(a_0, A_0)$, and

$$\begin{aligned}\rho V(a_0, A_0) &= u(c(a_0, A_0)) + \left(\frac{1-\tau_i}{1+\tau_k}(r_0 - \delta)a_0 - \tau_a a_0 + \frac{1-\tau_i}{1+\tau_c}w_0 - c(a_0, A_0)\right)V_a \\ &\quad + V_A A_0 \mu + \tfrac{1}{2}V_{AA}A_0^2\eta^2.\end{aligned} \tag{32}$$

is the maximized Bellman equation.

A.2 Explicit solutions

A.2.1 Proof of Theorem 2.1

The idea of this proof is to show that together with an educated guess of the value function, both the maximized Bellman equation (32) and first-order condition (31) are fulfilled. We may guess that the value function reads

$$V(a_t, A_t) = \frac{\phi^{-\sigma}a_t^{1-\sigma}}{1-\sigma} + f(A_t). \tag{33}$$

To start with we rewrite the policy function using the transformation in (30) as

$$C_t = \frac{1+\tau_k}{1+\tau_c}\phi K_t \ \Leftrightarrow\ Lc_t = \phi L a_t \ \Leftrightarrow\ c_t = \phi a_t. \tag{34}$$

Using (31) together with (5), and (34), we obtain $V_a = (\phi a_t)^{-\sigma}$. Moreover, our guess in (33) implies $V_A = f_A$, $V_{AA} = f_{AA}$. Inserting everything into (32) gives

$$\rho \frac{\phi^{-\sigma} a_t^{1-\sigma}}{1-\sigma} + g(A_t) = \frac{(\phi a_t)^{1-\sigma}}{1-\sigma} + (a_t\phi)^{-\sigma}\left(\frac{1-\tau_i}{1+\tau_k}(r_t-\delta)a_t - \tau_a a_t + \frac{1-\tau_i}{1+\tau_c}w_t - \phi a_t\right),$$

where we defined $g(A_t) \equiv \rho f(A_t) - f_A A_t \mu - \frac{1}{2} f_{AA} A_t^2 \eta^2$. Inserting factor rewards together with $K_t \equiv Lk_t = \frac{1+\tau_c}{1+\tau_k} La_t$ from (27), we obtain after some algebra,

$$\begin{aligned}\rho \frac{\phi^{-\sigma} a_t^{1-\sigma}}{1-\sigma} + g(A_t) &= \frac{(\phi a_t)^{1-\sigma}}{1-\sigma} + \frac{1-\tau_i}{1+\tau_c} A_t \left(\frac{1+\tau_c}{1+\tau_k}\right)^{\alpha} a_t^{\alpha}(a_t\phi)^{-\sigma} \\ &\quad - \frac{1-\tau_i}{1+\tau_k}\delta a_t (a_t\phi)^{-\sigma} - \tau_a a_t (a_t\phi)^{-\sigma} - \phi a_t (a_t\phi)^{-\sigma}.\end{aligned}$$

Using $\alpha = \sigma$ and $g(A_t) = \frac{1-\tau_i}{1+\tau_c} A_t \left(\frac{1+\tau_c}{1+\tau_k}\right)^{\alpha} \phi^{-\sigma}$ which pins down $f(\cdot)$, it reads

$$\begin{aligned}\rho \frac{\phi^{-\sigma} a_t^{1-\sigma}}{1-\sigma} &= \frac{(\phi a_t)^{1-\sigma}}{1-\sigma} - \frac{1-\tau_i}{1+\tau_k}\delta a_t^{1-\sigma}\phi^{-\sigma} - \tau_a a_t^{1-\sigma}\phi^{-\sigma} - (\phi a_t)^{1-\sigma} \\ \Leftrightarrow \rho &= \phi - (1-\sigma)\left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right) - (1-\sigma)\phi,\end{aligned}$$

which we finally can solve for ϕ in (10).

A.2.2 The evolution of capital rewards

Using Itô's formula (change of variables), capital rewards, $r_t = \alpha A_t K_t^{\alpha-1} L^{1-\alpha}$, follow

$$dr_t = r_A dA_t + r_K dK_t = r_t \mu dt + r_t \eta dB_t + r_K (I_t - \delta K_t) dt,$$

Now inserting $r_K = -(1-\alpha) r_t / K_t$, and replacing $Y_t/K_t = r_t/\alpha$, we obtain with (9)

$$\begin{aligned}dr_t &= r_t\mu dt + r_t\eta dB_t - (1-\alpha)(I_t/K_t - \delta) r_t dt, \\ &= \left(\mu - (1-\alpha)\left(\frac{1-\tau_i}{1+\tau_k} r_t/\alpha - \frac{1-\tau_i}{1+\tau_k}\delta - \tau_a - \frac{1+\tau_c}{1+\tau_k} C_t/K_t\right)\right) r_t dt + r_t \eta dB_t. \quad (35)\end{aligned}$$

A.2.3 Proof of Corollary 2.2

Inserting $C_t = \frac{1+\tau_k}{1+\tau_c}\phi K_t$ in the evolution of capital rewards (35), we obtain

$$dr_t = \left(\mu - (1-\alpha)\left(\frac{1-\tau_i}{1+\tau_k} r_t/\alpha - \frac{1-\tau_i}{1+\tau_k}\delta - \tau_a - \phi\right)\right) r_t dt + r_t\eta dB_t.$$

We now rewrite the equation by using the condition $\alpha = \sigma$, and inserting ϕ from (10) to

$$\begin{aligned}dr_t &= \left(\mu - \frac{1-\alpha}{\alpha}\left(\frac{1-\tau_i}{1+\tau_k} r_t - \rho - \left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)\right)\right) r_t dt + r_t \eta dB_t, \\ &= \frac{1-\alpha}{\alpha}\left(\frac{\alpha}{1-\alpha}\mu - \frac{1-\tau_i}{1+\tau_k} r_t + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a + \rho\right) r dt + r_t\eta dB_t \\ &= \frac{1-\alpha}{\alpha}\frac{1-\tau_i}{1+\tau_k} r_t \left(\frac{1+\tau_k}{1-\tau_i}\left(\frac{\alpha}{1-\alpha}\mu + \rho + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right) - r_t\right) dt + r_t\eta dB_t.\end{aligned}$$

Using the definitions c_1 and c_2 we finally obtain (11).

A.2.4 Proof of Corollary 2.3

Recall that the differential for log output is

$$d\ln Y_t = \left(\mu - \tfrac{1}{2}\eta^2 + \frac{1-\tau_i}{1+\tau_k}r_t - \alpha\frac{1+\tau_c}{1+\tau_k}C_t/K_t - \alpha\left(\tau_a - \frac{1-\tau_i}{1+\tau_k}\delta\right)\right)dt + \eta dB_t$$

Insert the policy function $C = \frac{1+\tau_k}{1+\tau_c}\phi K$ and use the condition $\alpha = \sigma$ gives for $y_t \equiv \ln Y_t$,

$$dy_t = \left(\mu + \alpha\left(\frac{1-\tau_i}{1+\tau_k}r_t/\alpha - \frac{1-\tau_i}{1+\tau_k}\delta - \tau_a - \phi\right) - \tfrac{1}{2}\eta^2\right)dt + \eta dB_t. \tag{36}$$

It denotes an affine SDE which solution is given by simple integration

$$y_t = y_{t_0} + (t-t_0)\left(\mu - \rho - \tau_a - \frac{1-\tau_i}{1+\tau_k}\delta - \tfrac{1}{2}\eta^2\right) + \frac{1-\tau_i}{1+\tau_k}\int_{t_0}^t r_s ds + \eta(B_t - B_0),$$

where r_t is known explicitly (cf. Posch 2009). Moreover, using the production function (1), log output is $y_t = \ln A_t + \alpha\ln K_t + (1-\alpha)\ln L$. Now insert the solution $C_t = \frac{1+\tau_k}{1+\tau_c}\phi K_t$ from Theorem 2.1 and define $\Delta y_t \equiv y_t - y_{t-\Delta}$, to obtain

$$\alpha(\ln C_t - \ln C_{t-\Delta}) = \Delta y_t - (\ln A_t - \ln A_{t-\Delta}) \tag{37}$$

Inserting the solutions to the SDEs in (2), $\ln A_t - \ln A_{t-\Delta} = (\mu - \frac{1}{2}\eta^2)\Delta + \eta(B_t - B_{t-\Delta})$, we obtain (12). Inserting the solution for log output (36) for $t_0 = t - \Delta$ we obtain

$$\begin{aligned}\alpha(\ln C_t - \ln C_{t-\Delta}) &= \left(\mu - \rho - \tau_a - \frac{1-\tau_i}{1+\tau_k}\delta - \tfrac{1}{2}\eta^2\right)\Delta + \frac{1-\tau_i}{1+\tau_k}\int_{t-\Delta}^t r_s ds \\ &\quad + \eta(B_t - B_{t-\Delta}) - \left(\left(\mu - \tfrac{1}{2}\eta^2\right)\Delta + \eta(B_t - B_{t-\Delta})\right) \\ &= -\left(\rho + \tau_a + \frac{1-\tau_i}{1+\tau_k}\delta\right)\Delta + \frac{1-\tau_i}{1+\tau_k}\int_{t-\Delta}^t r_s ds.\end{aligned}$$

which for $\alpha = \sigma$ is (13).

A.2.5 An alternative solution

The proofs for Theorem A.1, Corollaries A.2 and A.3 are analogous to Appendices A.2.1 to A.2.4 and available in a separate appendix on request from the author.

Theorem A.1 *If $\sigma > 1$ and the condition $\rho = (\alpha\sigma - 1)\left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right) - \sigma\mu + \frac{1}{2}(1+\sigma)\sigma\eta^2$ is fulfilled, consumption is a constant fraction of income, $C_t = \left(\frac{1+\tau_k}{1+\tau_c}\right)^\alpha \vartheta Y_t$, where*

$$\vartheta = \frac{\sigma - 1}{\sigma}\frac{1-\tau_i}{1+\tau_c}\left(\frac{1+\tau_c}{1+\tau_k}\right)^\alpha. \tag{38}$$

Corollary A.2 *The (before tax) rental rate of capital follows*

$$dr_t = c_3 r (c_4 - r_t) dt + \eta r_t dB_t \tag{39}$$

where $c_3 \equiv \frac{1-\alpha}{\alpha\sigma}\frac{1-\tau_i}{1+\tau_k}$, *and* $c_4 \equiv \frac{\alpha\sigma}{1-\alpha}\frac{1+\tau_k}{1-\tau_i}\mu + \alpha\sigma\frac{1+\tau_k}{1-\tau_i}\left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)$.

Corollary A.3 *The growth rate of output per unit of time,* $\Delta y_t \equiv y_t - y_{t-\Delta}$, *reads*

$$\Delta y_t = \left(\mu - \alpha\left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right) - \tfrac{1}{2}\eta^2\right)\Delta + 1/\sigma\frac{1-\tau_i}{1+\tau_k}\int_{t-\Delta}^{t} r_s ds + \eta(B_t - B_{t-\Delta}). \tag{40}$$

A.2.6 Moments of capital rewards

Consider the process in (11) or (39). Merton (1975) showed that r_t has a limiting Gamma distribution. Thus all moments exist and are known. Alternatively, we may use the following approach which is based on SDEs. Because $\ln r_t$ is a smooth transformation of r_t, the sequence $\{\ln r_t\}_{t=t_0}^{\infty}$ converges in distribution to a random variable $\ln r$,

$$\ln r_t \overset{\mathcal{D}}{\to} \ln r \quad \text{where} \quad -\infty < \ln r_t < \infty. \tag{41}$$

Compute the stochastic differential $d\ln r_t = c_1(c_2 - r_t)dt - \frac{1}{2}\eta^2 dt + \eta dB_t$. Because it describes a smooth transformation of r_t, it has a unique limiting distribution. If we apply expectations to the integral version, letting $t \to \infty$ we obtain

$$\begin{aligned}
\lim_{t\to\infty} E_0(\ln r_t) - \lim_{t\to\infty} E_0(\ln r_{t-\Delta}) &= (c_1c_2 - \tfrac{1}{2}\eta^2)\Delta - \lim_{t\to\infty}\int_{t-\Delta}^{t} c_1 E_0(r_s)ds \\
\Leftrightarrow \ 0 &= (c_1c_2 - \tfrac{1}{2}\eta^2)\Delta - \lim_{t\to\infty} c_1 E_0(r_t)\Delta \\
\Rightarrow \quad E(r) &\equiv \lim_{t\to\infty} E_0(r_t) = \frac{c_1c_2 - \frac{1}{2}\eta^2}{c_1}
\end{aligned} \tag{42}$$

which is the asymptotic mean, i.e., the mean of the limiting distribution.

Similarly, using the integral version of (11), asymptotically

$$\begin{aligned}
\lim_{t\to\infty} E_0(r_t) - \lim_{t\to\infty} E_0(r_{t-\Delta}) &= \lim_{t\to\infty}\int_{t-\Delta}^{t}(c_1c_2)E_0(r_s)ds - \lim_{t\to\infty}\int_{t-\Delta}^{t} c_1 E_0(r_s^2)ds \\
\Leftrightarrow \ 0 &= \lim_{t\to\infty}(c_1c_2)E_0(r_t)\Delta - \lim_{t\to\infty} c_1 E_0(r_t^2)\Delta \\
\Rightarrow \quad E(r^2) &\equiv \lim_{t\to\infty} E_0(r_t^2) = E(r)c_2.
\end{aligned}$$

The variance of the limiting distribution is $Var(r) = E(r^2) - E(r)^2 = E(r)\frac{1}{2}\eta^2/c_1$.

A.3 Volatility and taxation

A.3.1 A stochastic balanced growth property

Lemma A.4 *Given the restriction* $\alpha = \sigma$, $Cov(\Delta c_t, B_t - B_{t-\Delta}) = 0$.

Proof. Inserting $C_t = \frac{1+\tau_k}{1+\tau_c}\phi K_t$ from Theorem 2.1, we have to show that

$$0 \;=\; Cov\left(\ln K_t - \ln K_{t-\Delta}, B_t - B_{t-\Delta}\right) = Cov\left(\int_{t-\Delta}^{t} d(\ln K_s), \int_{t-\Delta}^{t} dB_s\right)$$

which holds because from (3) $d\ln K_t = (I_t/K_t - \delta)dt$ is instantaneously deterministic. ■

We are prepared to compute moments of output growth rates. Using (12), we obtain

$$E(\Delta y_t) \equiv \lim_{t\to\infty} E_0(\Delta y_t) \;=\; \left(\mu - \tfrac{1}{2}\eta^2\right)\Delta + \sigma \lim_{t\to\infty} E_0(\Delta c_t) \tag{43}$$

where from (13)

$$\begin{aligned} \sigma \lim_{t\to\infty} E_0(\Delta c_t) &= \lim_{t\to\infty} \frac{1-\tau_i}{1+\tau_k}\int_{t-\Delta}^{t} E_0(r_s)ds - \left(\rho + \tau_a + \frac{1-\tau_i}{1+\tau_k}\delta\right)\Delta \\ &= \frac{1-\tau_i}{1+\tau_k}E(r)\Delta - \left(\rho + \tau_a + \frac{1-\tau_i}{1+\tau_k}\delta\right)\Delta = \frac{\alpha}{1-\alpha}(\mu\Delta - \tfrac{1}{2}\eta^2\Delta), \end{aligned}$$

using (42) in the last step. Plugging the result back into (43), we obtain the unconditional mean of output growth rates in (14). Observe that consumption and output asymptotically grow at the same exogenous rate, which is the stochastic equivalent to the balanced growth property in the standard neoclassical model.

A.3.2 Moments of integrated capital rewards

For the variance of output growth rates, we use (12) and Lemma A.4 to obtain (15), which shows it is given by the variance of consumption growth rates and the variance of the shocks. To compute the variance of consumption growth rates, we start from (13),

$$\lim_{t\to\infty} \sigma^2 Var_0(\Delta c_t) = \lim_{t\to\infty} Var_0\left(\frac{1-\tau_i}{1+\tau_k}\int_{t-\Delta}^{t} r_s ds\right) \tag{44}$$

where the asymptotic variance of the integrated process (11) can be written as

$$\lim_{t\to\infty} Var\left(\int_{t-\Delta}^{t} r_s ds\right) = \lim_{t\to\infty} E_0\left(\int_{t-\Delta}^{t}\int_{t-\Delta}^{t} r_s r_u ds du\right) - \lim_{t\to\infty}\left(\int_{t-\Delta}^{t} E_0(r_s)ds\right)^2 \tag{45}$$

Therefore, to compute the variance of the integrated process, we need joint moments $E(r_s r_u)$. Consider $s > u$, then $E(r_s r_u) = E(r_u E(r_s|r_u))$. We use the deterministic Taylor expansion to compute the conditional expectation (e.g. Aït-Sahalia 2008),

$$E(g(r_s)|r_u) = \sum_{i=0}^{k} \frac{\Delta^i}{i!} A^i g(r_u) + O(\Delta^{k+1}) \tag{46}$$

where A is the infinitesimal generator of the process, $Ag(x) = c_1 x(c_2 - x)g'(x) + \frac{1}{2}c_3^2 x^2 g''(x)$. The function $g(\cdot)$ is sufficiently differentiable. As a result, we obtain a closed-form expansion of the conditional expectation around r_u. Using $g(x) = x$, we obtain

$$Ax = c_1 x(c_2 - x), \quad A^2 x = c_1^2 c_2 x(c_2 - x) - c_1(c_1 x(c_2 - x)2x + \tfrac{1}{2}c_3^2 x^2 2).$$

Using a second-order Taylor expansion (46), the conditional expectation reads

$$E(r_s|r_u) \;=\; r_u + (c_1c_2r_u - c_1r_u^2)\Delta + \left(\tfrac{1}{2}c_1^2c_2^2r_u - c_1(\tfrac{3}{2}c_1c_2 + \tfrac{1}{2}c_3^2)r_u^2 + c_1^2r_u^3\right)\Delta^2 + O(\Delta^3)$$

Hence, we obtain for the joint moments

$$\begin{aligned} E(r_sr_u) = E(r_uE(r_s|r_u)) \;&=\; E(r_u^2) + (c_1c_2E(r_u^2) - c_1E(r_u^3))\Delta \\ &\quad + \left(\tfrac{1}{2}c_1^2c_2^2E(r_u^2) - c_1(\tfrac{3}{2}c_1c_2 + \tfrac{1}{2}c_3^2)E(r_u^3) + c_1^2E(r_u^4)\right)\Delta^2 + O(\Delta^3) \end{aligned}$$

and the variance of the integrated process in (45) reads

$$\begin{aligned} \lim_{t\to\infty} Var\left(\int_{t-\Delta}^{t} r_s ds\right) \;&=\; E(r_sr_u)\Delta^2 - E(r)^2\Delta^2 \\ &=\; Var(r)\Delta^2 + (c_1c_2E(r^2) - c_1E(r^3))\Delta^3 \\ &\quad + \left(\tfrac{1}{2}c_1^2c_2^2E(r^2) - c_1(\tfrac{3}{2}c_1c_2 + \tfrac{1}{2}c_3^2)E(r^3) + c_1^2E(r^4)\right)\Delta^4 + O(\Delta^5) \end{aligned}$$

Plugging the result back into (44) gives the variance of consumption growth in (16). Finally, neglecting third-order terms, and inserting the asymptotic variance of capital rewards as from Appendix A.2.6, gives our measure in (17).[33]

A.4 Data appendix

A.4.1 Data sources

We use the following databases from OECDiLibrary (http://www.oecd-ilibrary.org/):

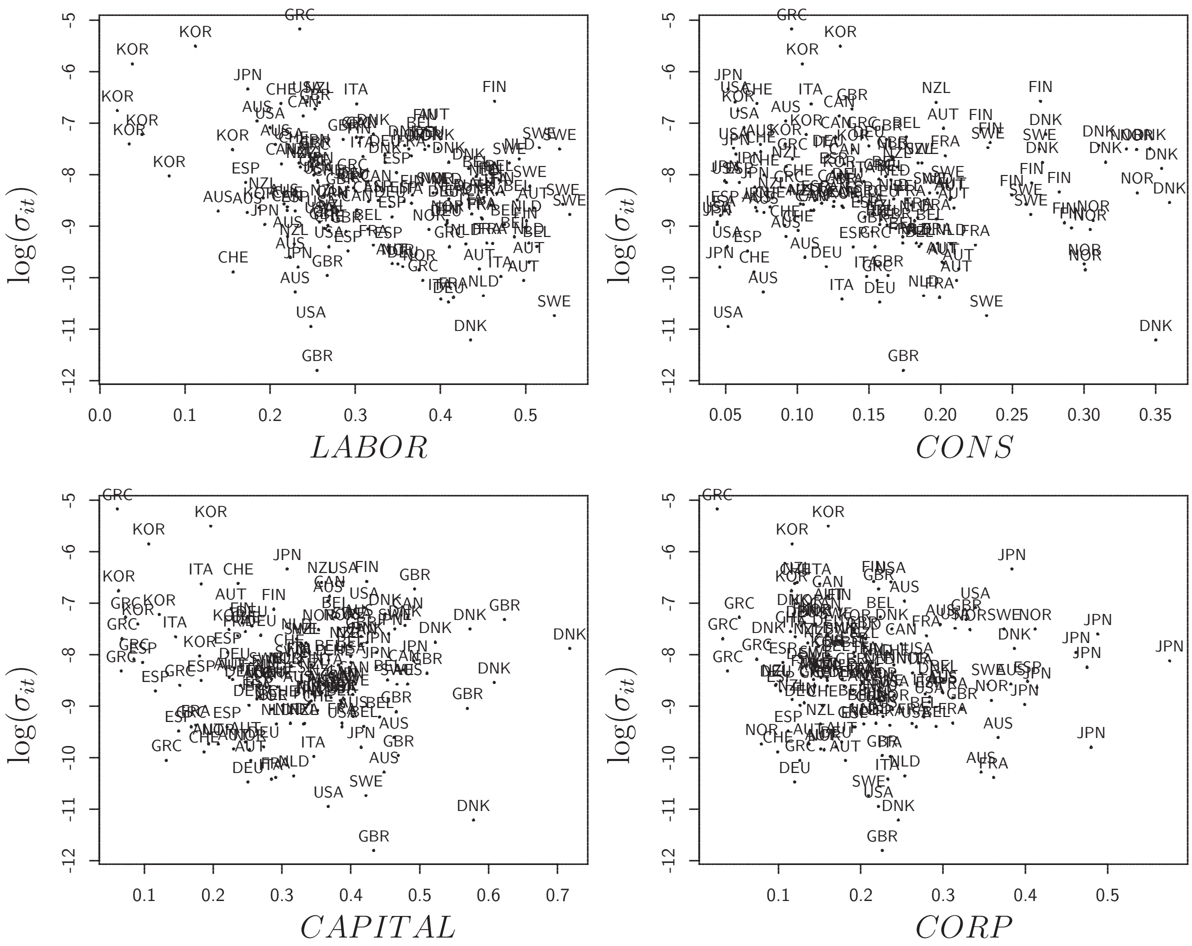
- Revenue Statistics (2010, http://dx.doi.org/10.1787/data-00262-en)
- Detailed National Accounts (2010, http://dx.doi.org/10.1787/na-dna-data-en)
- Aggregate National Accounts (2010, http://dx.doi.org/10.1787/na-ana-data-en)
- General Government Accounts (2010, http://dx.doi.org/10.1787/data-00020-en)
- Quarterly National Accounts (2010, http://dx.doi.org/10.1787/data-00017-en)
- Main Economic Indicators (2010, http://dx.doi.org/10.1787/data-00052-en)

The tax ratios based on Mendoza et al. (1994) are available on request, which may be used without further permission provided that full credit is given to the source.

A.5 Empirical results

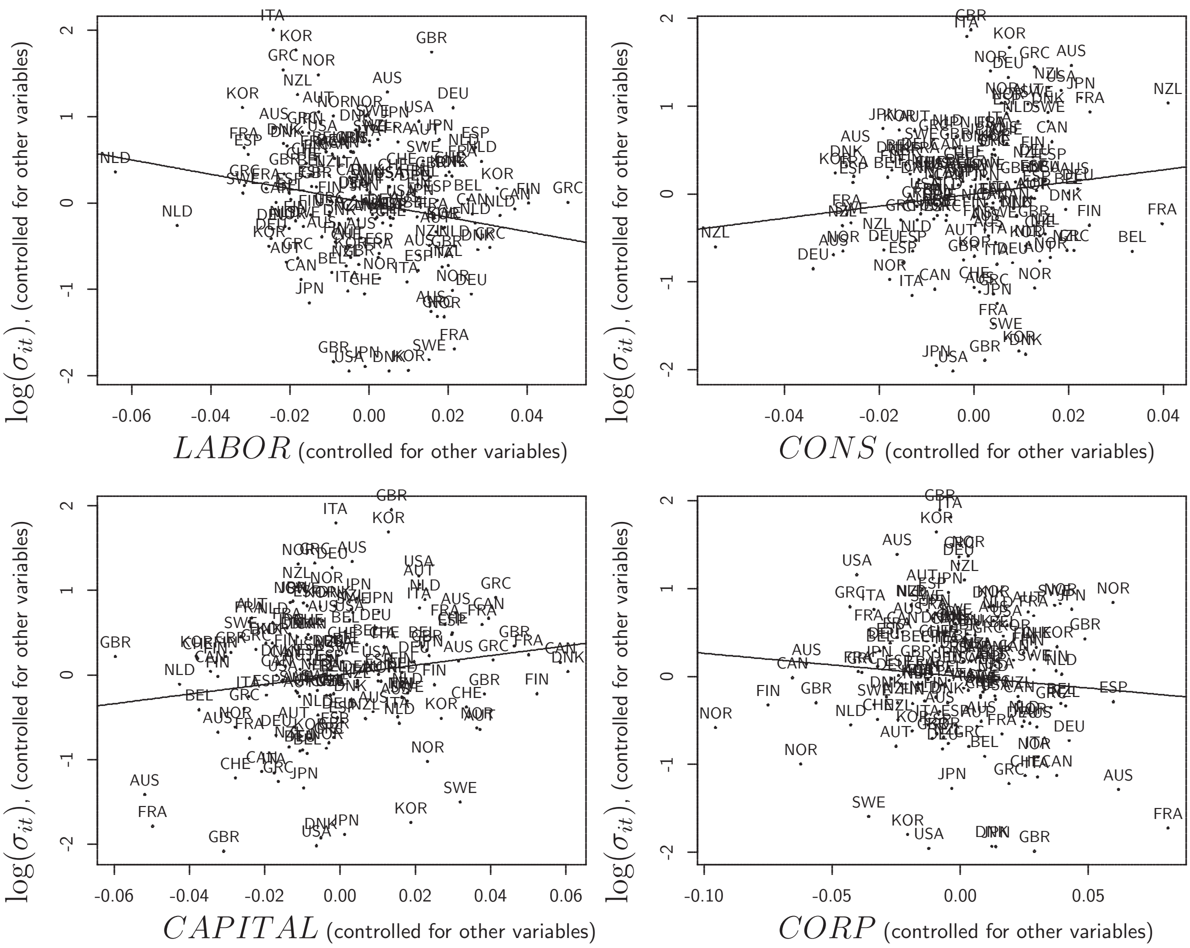
[33]The author thanks Michael Sørensen for helpful discussions on this issue.

Figure A.1: Simple correlation of taxes and *observed* volatility (fixed-windows)



Notes: These figures give scatter plots of *observed* volatility measured as log of the variance of annual growth rates of real GDP per capita against tax rates using the fixed-window (five-year) panel approach.

Figure A.2: Partial correlation of taxes and *observed* volatility (fixed-windows)



Notes: These figures illustrate tax effects on *observed* volatility measured as the log of the variance of annual growth rates of real GDP per capita using fixed-windows (five-year) controlling for other variables (cf. Table 4, column 2).

Table A.1: Static panel estimation, *observed* volatility measures (five-year windows)

OECD		IWLS (fixed-window)	IWLS (fixed-window)	IWLS (rolling-window)	IWLS (rolling-window)
$LABOR_{it}$	β_1	−4.72 (2.86)	−6.84 (2.98) *	−7.51 (1.37) ***	−7.21 (1.56) ***
$CAPITAL_{it}$	β_2	5.81 (2.63) *	4.71 (2.47) ·	7.33 (1.24) ***	4.14 (1.35) **
$CONS_{it}$	β_3	8.00 (3.95) *	6.68 (3.75) ·	7.66 (2.01) ***	10.10 (2.13) ***
$CORP_{it}$	β_4	−2.47 (2.18)	−1.77 (2.02)	−5.55 (0.99) ***	−3.44 (1.07) **
$GROW_{it}$	γ_1		−20.02 (7.18) **		−10.10 (1.91) ***
$PRIVY_{it}$	γ_2		0.16 (0.34)		−0.18 (0.19)
$INFL_{it}$	γ_3		−14.02 (3.05) ***		0.59 (1.41)
$INFLSD_{it}$	γ_4		26.34 (5.23) ***		8.70 (3.34) **
$GGDP_{it}$	γ_5		−5.00 (6.04)		2.09 (3.40)
$GGDPSD_{it}$	γ_6		34.10 (24.17)		32.89 (12.46) **
$XRSD_{it}$	γ_7		1.13 (2.97)		3.49 (1.77) ·
$OPEN_{it}$	γ_8		1.54 (1.03)		0.75 (0.55)
Degrees of freedom		129	120	669	624
Country fixed effects	α_i	yes	yes	yes	yes
Time fixed effects	λ_t	yes	yes	yes	yes

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '·' 0.1 ' ' 1

Notes: This table reports the semi-elasticities of the fixed-effects model (20) using iterated weighed least squares estimation (IWLS), explaining the variance of annual growth rates of real GDP per capita. Standard errors of $R = 4999$ model-based bootstrap replicates using the adjusted percentile method are in parentheses. We use one-period lagged controls for the rolling window approach.

Table A.2: Dynamic panel estimation, *observed* variances (fixed-windows)

OECD		LSDV (taxes only)	LSDV (full model)	LSDV (no time effects)	LSDV (no time effects)
$\Delta LABOR_{i,t}$	ϑ_1	-11.62 (4.39) **	-14.87 (4.59) **	-15.15 (4.77) **	-17.08 (4.86) ***
$\Delta CAPITAL_{i,t}$	ϑ_2	4.42 (3.75)	0.50 (3.86)	6.12 (4.08)	0.07 (4.06)
$\Delta CONS_{i,t}$	ϑ_3	10.03 (6.74)	17.11 (6.93) *	-2.08 (6.57)	2.49 (6.35)
$\Delta CORP_{i,t}$	ϑ_4	-4.43 (3.01)	-0.96 (3.07)	-5.26 (3.20)	-0.07 (3.24)
$LABOR_{i,t-1}$	β_1	-7.87 (3.25) *	-8.13 (3.94) *	-15.42 (2.84) ***	-12.87 (3.69) ***
$CAPITAL_{i,t-1}$	β_2	10.10 (3.10) **	6.98 (3.42) *	10.71 (3.22) **	6.12 (3.34) ·
$CONS_{i,t-1}$	β_3	12.31 (4.63) **	9.76 (5.49) ·	3.01 (4.37)	7.28 (5.29)
$CORP_{i,t-1}$	β_4	-5.37 (2.57) *	-2.88 (2.86)	-5.21 (2.56) *	-1.83 (2.78)
$GROW_{i,t}$	γ_1		-21.96 (9.32) *		-26.09 (8.79) **
$PRIVY_{i,t}$	γ_2		0.13 (0.46)		-0.65 (0.42)
$INFL_{i,t}$	γ_3		-10.93 (4.92) *		0.06 (4.61)
$INFLSD_{i,t}$	γ_4		19.37 (8.50) *		14.53 (8.95)
$GGDP_{i,t}$	γ_5		-5.53 (8.28)		5.04 (8.22)
$GGDPSD_{i,t}$	γ_6		56.81 (33.90) ·		17.00 (33.96)
$XRSD_{i,t}$	γ_7		0.81 (3.86)		-2.05 (3.89)
$OPEN_{i,t}$	γ_8		-0.32 (1.46)		-0.25 (1.18)
$\sigma_{i,t-1}$	ρ	-0.21 (0.09) *	-0.12 (0.09)	-0.25 (0.09) **	-0.21 (0.08) *
Country fixed effects	α_i	yes	yes	yes	yes
Time fixed effects	λ_t	yes	yes	no	no
Degrees of freedom		105	96	111	102

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '·' 0.1 ' ' 1

Notes: This table reports the semi-elasticities of the *dynamic* model specification (24) using the non-linear least square dummy variable approach (LSDV) explaining the variance of annual growth rates of real GDP per capita. We also estimate the linear model (not shown) using the reduced form parameter $(\rho-1)\beta$ and apply the bias correction for dynamic panel models suggested by Bun and Carree (2005). Since we find a negligible bias we present the non-linear estimation results.

Table A.3: Transitional dynamics in the growth equation

OECD		MLE (taxes only)	MLE (full model)	MLE (no time effects)	MLE (no time effects)
$LABOR_{i,t-1}$	δ_1	0.05 (0.02) ·	0.04 (0.02) ·	0.05 (0.03) ·	0.04 (0.02) ·
$CAPITAL_{i,t-1}$	δ_2	−0.07 (0.03) **	−0.06 (0.03) *	−0.08 (0.03) *	−0.07 (0.03) *
$CONS_{i,t-1}$	δ_3	0.03 (0.04)	0.03 (0.04)	0.03 (0.04)	0.02 (0.04)
$CORP_{i,t-1}$	δ_4	0.03 (0.02)	0.03 (0.02)	0.02 (0.02)	0.02 (0.02)
$GROW_{i,t-1}$	φ_1	0.37 (0.04) ***	0.41 (0.04) ***	0.34 (0.04) ***	0.41 (0.05) ***
$GROW_{i,t-2}$	φ_2	−0.09 (0.04) *	−0.09 (0.04) *	−0.11 (0.04) *	−0.12 (0.04) **
$LABOR_{i,t-1}$	β_1	−7.36 (2.84) **	−5.07 (3.46)	−15.09 (1.99) ***	−9.95 (2.83) ***
$CAPITAL_{i,t-1}$	β_2	6.70 (2.50) **	5.59 (2.56) *	7.72 (2.26) ***	5.29 (2.35) *
$CONS_{i,t-1}$	β_3	6.31 (3.94)	8.11 (4.15) ·	−1.28 (3.50)	2.76 (3.97)
$CORP_{i,t-1}$	β_4	−1.84 (2.13)	−0.97 (2.13)	−2.33 (1.83)	−0.18 (1.85)
$GROW_{i,t-1}$	γ_1		−13.47 (4.08) **		−14.60 (3.84) ***
$PRIVY_{i,t-1}$	γ_2		−0.17 (0.37)		−0.45 (0.24) ·
$INFL_{i,t-1}$	γ_3		−0.78 (2.77)		2.13 (2.49)
$INFLI_{i,t-1}$	γ_4		11.28 (5.33) *		13.57 (4.98) **
$GGDP_{i,t-1}$	γ_5		−14.28 (6.87) *		−7.50 (6.21)
$DGFI_{i,t-1}$	γ_6		−0.41 (4.02)		−1.08 (3.90)
$XRFI_{i,t-1}$	γ_7		1.64 (2.63)		1.79 (2.53)
Degrees of freedom		636	629	642	635
Log-likelihood		−1853.4	−1868.1	−1831.5	−1854.6
Country fixed effects	θ_i	yes	yes	yes	yes
Country fixed effects	α_i	yes	yes	yes	yes
Time fixed effects	λ_t	yes	yes	no	no

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '·' 0.1 ' ' 1

Notes: This table reports the semi-elasticities of the joint estimation of (25) using maximum likelihood, explaining the conditional variance of annual growth rates of real GDP per capita. Asymptotic standard errors are in parentheses.