On the link between volatility and growth

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Abstract

A model of growth with endogenous innovation and distortionary taxes is presented. Since innovation is the only source of volatility, any variable that influences innovation directly affects volatility and growth. This joint endogeneity is illustrated by working out the effects through which economies with different tax *levels* differ in their volatility and growth process. We obtain analytical measures of macro volatility based on cyclical output and on output growth rates for plausible parametric restrictions. This analysis implies that controls for taxes should be included in the standard growth-volatility regressions. Our estimates show that the conventional Ramey-Ramey coefficient is affected sizeably. In addition, tax *levels* do indeed appear to affect volatility in our empirical application.

Keywords: Tax effects, Volatility measures, Poisson uncertainty, Endogenous cycles and growth, Continuous-time DSGE models
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1 Introduction

Background. In a seminal paper, Ramey and Ramey (1995) find a strong empirical negative link between volatility and growth. Subsequent papers confirm this relationship for other datasets (Martin and Rogers, 1997; Acemoglu, Johnson, Robinson and Thaicharoen, 2003; Aghion, Angeletos, Banerjee and Manova, 2010; Posch, 2009a).[1] Complementary to Ramey and Ramey, these studies use additional controls such as exchange rate variability, financial development and measures of openness, institutions or monetary and fiscal policy.

The open question. The previous studies are primarily of an empirical nature. These papers do not, however, inquire into the exact structural channels through which macro volatility and growth interact. It therefore largely remains an open question what the determinants of the link between volatility and growth are.

Our message. This paper plays the devil's advocate and argues that, theoretically, any correlation pattern can emerge. The sign of the growth-volatility correlation depends – inter alia – on time-invariant economic policies of a country. We show that a link between volatility and growth may arise from endogenous innovations and their propagation. Volatility is endogenous for measures based on cyclical output and for measures based on output growth rates. Growth emerges endogenously from innovations of a research sector. In our theoretical framework the *level* of taxes jointly affects growth and volatility and thereby their link.

Our framework. Our analysis builds on dynamic stochastic equilibrium models in which cyclical growth emerges endogenously (Bental and Peled, 1996; Matsuyama, 1999; Francois and Lloyd-Ellis, 2003, 2008; Wälde, 2002, 2005).[2] We use a version of Wälde (2005), because it has analytical solutions for plausible parametric restrictions for non-trivial dynamics and comprises the continuous-time real business cycle (RBC) model as a special case.

Using a well-known parametric restriction, we obtain two types of analytical volatility measures (borrowing extensively from García and Griego, 1994). The first type is based on stochastically detrended (henceforth cyclical) variables. The second type is based on growth rates of the original (non-stationary) series. While the first one is identical in spirit to empirical decompositions, where a time series is split into a growth trend and a stationary cyclical component, the second one is the common measure in the empirical literature.

Results. We illustrate how structural parameters affect our measures of volatility and growth directly by changing the variance and intensity of the shocks, and indirectly by affecting the shock propagation. In our model, we focus on taxes as an example for economic

[1] There is work suggesting that the link is not as pronounced when using time series evidence (Beaumont, Norrbin and Yigit, 2008). At different levels of aggregation either no significant relationship is found using state data (Dawson and Stephenson, 1997), or an ambiguous empirical result – either a positive or negative link – is found at the sectoral level (Imbs, 2007; Chong and Gradstein, 2009).

[2] These papers in turn build on Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom, Anant and Dinopoulos (1990). The present paper builds explicitly on the stochastic Aghion and Howitt model using the formulation for risk averse agents introduced by Wälde (1999).

policy parameters. Any correlation between volatility and growth can be predicted if the growth and volatility measures in our model economy are considered for different tax levels. The volatility-growth link can even change sign if tax rates are altered. We identify three channels through which macro volatility can be affected by policy parameters, i.e., the speed of convergence (to the non-stochastic steady state), the jump size and the arrival rate.

For empirical studies of the Ramey-Ramey type, the inclusion of control variables in the conditional variance equation is suggested. By including such controls, we find that 3 out of the 4 controls we suggest are significant at the 5% level. We also find that the conventional Ramey-Ramey coefficient which links volatility and growth is affected sizably.

Further related literature. Our theoretical and empirical results are complementary to the finding that volatility can have detrimental effects on growth (Chong and Gradstein, 2009). The authors provide empirical evidence that unexpected changes in economic and fiscal policies is a channel through which the link between volatility and growth could materialize at the macro level. We show that even with constant policies the link materializes through the joint endogeneity. Jaimovic and Siu (2009) also provide evidence for controlling for additional variables in Ramey-Ramey regressions. While they do stress the effect of the age composition of the labor force on volatility, they do not estimate the link between volatility and growth. Aghion et al. (2010) argue that tighter credit constraints can lead to both lower and more volatile growth rates. While this is complementary to our channel, they do not study the implications for the conventional Ramey-Ramey coefficient.

Table of contents. The paper proceeds as follows. Section 2 introduces the model of endogenous innovation and distortionary taxes. Section 3 presents the equilibrium dynamics and illustrates the notion of cyclical growth. Section 4 contains our theoretical contribution, the derivation of closed-form volatility measures. Section 5 comprises our main economic insights and section 6 provides our empirical results. The final section concludes.

2 The model

Production possibilities. Technological progress is labor augmenting and embodied in capital. All capital goods can be identified by a number denoting their date of manufacture and therefore their vintage. A capital good K_j of vintage j allows workers to produce with labor productivity A^j, where $A > 1$ is a constant parameter. Hence, a more modern vintage $j+1$ implies a labor productivity that is A times higher than that of vintage j. The corresponding production function reads $Y_j = K_j^\alpha (A^j L_j)^{1-\alpha}$, where the amount of labor allocated to that vintage is L_j and $0 < \alpha < 1$ denotes the output elasticity of capital.

There is a very large number of research firms which operate under perfect competition. Research costs are recovered by returns of a prototype which is the outcome of a successful project. This differs from standard modeling of R&D where successful research only leads to a blueprint. The prototype is a production unit – a machine – of size κ_t. This new prototype is owned by the individuals who financed the successful R&D project (as reflected in the budget constraint below). The currently most advanced vintage is denoted by q and implies a labor productivity of A^q.[3] The new prototype yields a labor productivity of A^{q+1} for workers having access to this new technology.

Research is a risky activity. Uncertainty in research is captured by a Poisson process q_t where the arrival rate of success is denoted by λ_t. Resources employed for research are denoted by R_t. An exogenous function D_t captures the difficulty to make an invention (as in Segerstrom, 1998). This function captures the idea that an economy needs to put more effort into research for the next generation of capital goods if new technologies are to appear at a constant rate. There are constant returns to scale at the firm level. On the sectorial level, however, an externality $h(\cdot)$ implies decreasing returns to scale,

$$\lambda_t = (R_t/D_t)h(R_t/D_t) \equiv (R_t/D_t)^{1-\gamma}, \quad 0 < \gamma < 1, \tag{1}$$

where the difficulty function D_t and the externality $h(\cdot)$ are taken as given by the firm.[4]

Given this research process, the capital stock of the next vintage follows

$$dK_{q+1} = \kappa_t dq_t, \tag{2}$$

which is a simple stochastic differential equation (SDE). The increment dq_t of the Poisson process q_t can either be 0 or 1. As successful research means $dq_t = 1$, this equation states that the capital stock increases from 0 to κ_t in the good outcome. When research is not successful, $dK_{q+1} = 0$ because $dq_t = 0$.

Capital accumulation of existing vintages 1 to q is riskless. When resources are used to accumulate existing capital, the capital stock of vintage j increases if investment in vintage j exceeds depreciation δ,

$$dK_j = (I_j - \delta K_j)\, dt, \quad j \le q. \tag{3}$$

Given that value marginal productivity is highest for the most advanced vintage, investment only takes place in vintage q. As R&D takes place under perfect competition, there is no monopolist owning the new vintage and there is no patent protection. Thus, we observe $I_j = 0\ \forall\ j < q$, and $I_q = I_t$ for the most advanced vintage. As soon as a new capital good is discovered through R&D, it is replicated by a large number of competing firms. In contrast to R&D, this is a deterministic process because capital accumulation simply means replicating

[3] More precisely, q_t denotes the Poisson process whereas q denotes the label of the most recent vintage (number of jumps up to time t). Though in principle interchangeable, after successful research, q_t increases by 1 while the label of older vintages remains like a stamp on the capital goods.

[4] Remember that arrival rates of Poisson processes can be added. Economically speaking, this means that there are many "small" arrival rates $\lambda_t^f = (R_t^f/D_t)h(R_t/D_t)$ where R_t^f stands for R&D investment in research firm f. Aggregating over all research firms leads to the economy wide arrival rate λ_t.

existing machines. The process of capital accumulation is also – as in the standard Solow growth model – perfectly competitive.

Before we continue with the description of the model, we present a few equilibrium properties, some of them related to the vintage capital structure used here. They are useful as they simplify the presentation of the government, preferences and the assumptions about the difficulty function as well as the size of the prototype. Each vintage of capital allows a single output good to be produced, which is used for producing consumption goods, C_t, investment goods, I_t, as an input for research, R_t, and for government expenditures, G_t,

$$\sum_{j=0}^{q} Y_j \;=\; Y_t \;=\; C_t + I_t + R_t + G_t, \tag{4}$$

where the quantities denote *net* resources used for these activities, i.e., after taxation. All activities in the economy take place under perfect competition. Hence, the producer price of the production good, the consumption good, and both investment goods used for capital accumulation and research will therefore be identical,

$$p_t^Y = p_t^C = p_t^K = p_t^R. \tag{5}$$

Aggregate constant labor supply in this economy is L. Allowing labor to be mobile across all vintages such that wage rates equalize and assuming market clearing, $\sum_{j=0}^{q} L_j = L$, total output of the economy can be represented by a simple Cobb-Douglas production function,

$$Y_t = K_t^{\alpha} L^{1-\alpha}, \tag{6}$$

in which vintage-specific capital has been combined to an aggregate capital index K_t,

$$K_t = K_0 + BK_1 + ... + B^q K_q = \sum_{j=0}^{q} B^j K_j, \quad B \equiv A^{\frac{1-\alpha}{\alpha}}. \tag{7}$$

This index can be thought of as counting the 'number of machines' of the first vintage, $j = 0$, that would be required to produce the same output Y_t as with the current mix of vintages.

Applying Itô's formula (or change of variable formula, cf. Sennewald (2007) for a rigorous analysis and Sennewald and Wälde (2006) for an introduction) to (7) using (2) and (3), the capital index K_t follows the SDE,

$$dK_t = (B^q I_t - \delta K_t)\, dt + B^{q+1} \kappa_t dq_t. \tag{8}$$

Because the capital index, K_t, is measured in units of the first vintage, it increases as a function of effective investment, $B^q I_t$, minus depreciation, δK_t. When an innovation occurs, the capital index increases by the effective size of the new prototype, $B^{q+1}\kappa_t$.

Government. The government levies taxes on income, τ_i, on wealth, τ_a, on consumption expenditures, τ_c, on investment expenditures, τ_k, and on research expenditures, τ_r. In our study, a positive tax either implies a real decrease in income or an increase in the effective price (consumer price), whereas a negative tax denotes a subsidy. The government uses all tax income (and does not save or run a debt) to provide basic government services G_t,

$$G_t = \tau_i(Y_t - \delta B^{-q}K_t) + \tau_k(I_t - \delta B^{-q}K_t) + \tau_r R_t + \tau_c C_t + \tau_a(1+\tau_k)B^{-q}K_t \geq 0. \quad (9)$$

In order to focus on the effects of taxation from government expenditures, we assume that government expenditure does not affect household utility or the production possibilities of the economy. A myopic government simply provides basic government services without having any interest in stabilization policy or optimal taxation. The tax structure thus is exogenously given to the model. Additional effects through the channel of fiscal debt might be interesting but beyond the scope of this paper.

Producer prices from (5) are identical for all three production processes. When goods are sold, they are taxed differently such that consumer prices are $(1+\tau_c)p_t^C$, $(1+\tau_k)p_t^K$, $(1+\tau_r)p_t^R$, respectively. To rule out arbitrage between different types of goods, we assume that a unit of production is useless for other purposes once it is assigned for a special purpose: once a consumption good is acquired, it cannot be used for, e.g., capital accumulation.

Sales taxes have no theoretical upper bound. A 300% tax on the consumption good would imply that 3/4 of the price are taxes going to the government and 1/4 goes to the producer. Their lower bound is clearly -100%, when the good would be gratis. Similarly, the upper bound for taxes on income is 100% (instant confiscation of income), while there is no lower bound. Hence, we obtain $-1 < \tau_c, \tau_k, \tau_r$ and $\tau_i, \tau_a < 1$.

Preferences. The economy has a large number of representative households. Households maximize expected utility given by the integral over instantaneous utility, $u = u(c_t)$, resulting from consumption flows, c_t, and discounted at the subjective rate of time preference, ρ,

$$U_0 = E_0 \int_0^\infty e^{-\rho t} u(c_t) dt. \quad (10)$$

We assume that instantaneous utility is characterized by constant relative risk aversion,

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad \sigma > 0. \quad (11)$$

The budget constraint reflects investment possibilities in this economy, the impact of taxes and shows how real wealth, a_t, evolves over time. Households can invest in a risky asset by financing research, i_t, and in an (instantaneously) riskless asset by replicating capital. We measure wealth in units of the consumption good, priced at consumer prices. The household's budget constraint can best be illustrated by looking at

$$\begin{aligned} da_t &= \left(\frac{1-\tau_i}{1+\tau_c}\sum_{j=0}^{q+1} w_j^K k_j + \frac{1-\tau_i}{1+\tau_c} w_t - c_t - \frac{1+\tau_r}{1+\tau_c} i_t\right) dt - \left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right) a_t dt \\ &\quad + \left(\frac{1+\tau_k}{1+\tau_c}\kappa_t \frac{i_t}{R_t} - \frac{B-1}{B} a_{t-}\right) dq_t, \end{aligned} \quad (12)$$

where $a_{t-} \equiv \lim_{s \to t} a_s$, $s < t$, denotes individual wealth an instant before a jump in t. The first sum in (12), $\sum_{j=0}^{q+1} w_j^K k_j$, captures capital income from all vintages. This is taxed at the income tax rate τ_i and divided by the after-tax price of the consumption good, $1 + \tau_c$ (keeping in mind that the consumption good is the numeraire). Hence, the entire term $\frac{1-\tau_i}{1+\tau_c} \sum_{j=0}^{q+1} w_j^K k_j$ captures after-tax capital income in units of the consumption good. The same reasoning applies to labor income w_t, consumption expenditures c_t, and investment into research i_t. Thus, the first bracket captures the increase in wealth a_t measured in units of the consumption good at after-tax consumer prices. The second term captures the deterministic wealth-reducing effect due to depreciation and the tax on wealth, where the tax rates in front of the depreciation rate ensure that only net capital rewards (after depreciation) are taxed. The third term is a stochastic component which increases the individual's wealth in case of successful research by the 'dividend payments' less 'economic depreciation'. Here, 'dividend payments' at the household level are given by the share i_t/R_t of a successful research project financed by the household times total payoffs determined by the size κ_t of the prototype times its value in units of the consumption good, i.e., $\frac{1+\tau_k}{1+\tau_c}$. The term $1 + \tau_k$ points out that a successful research project yields an installed capital good (and not an investment good). Moreover, 'economic depreciation' of $s \equiv \frac{B-1}{B} > 0$ percent emerges from the vintage capital structure as the most advanced vintage from (5) has a relative price of unity and all other vintages lose in value relative to the consumption good.

After some algebra, the budget constraint can be written as follows (cf. app. A.2),[5]

$$\begin{aligned} da_t &= \left(\left(\frac{1-\tau_i}{1+\tau_k}(r_t - \delta) - \tau_a \right) a_t + \frac{1-\tau_i}{1+\tau_c} w_t - c_t - \frac{1+\tau_r}{1+\tau_c} i_t \right) dt \\ &\quad + \left(\frac{1+\tau_k}{1+\tau_c} \kappa_t \frac{i_t}{R_t} - s a_{t-} \right) dq_t, \end{aligned} \tag{13}$$

where factor rewards

$$r_t = B^q \frac{\partial Y_t}{\partial K_t} \equiv B^q Y_K, \quad w_t = \frac{\partial Y_t}{\partial L} \equiv Y_L, \tag{14}$$

are defined by the rental rate of capital and the wage rate respectively.

Assumptions. For the problem to be well defined, we need assumptions on the functional forms of the 'difficulty function' as well as on the 'size' of the new prototype. We capture the innovations of the past by the current (tax-independent) size of total wealth, $K_t^{obs} = La_t$,

$$D_t \equiv D \frac{1+\tau_c}{1+\tau_k} K_t^{obs} = DB^{-q} K_t, \quad D > 0. \tag{15}$$

Measuring wealth in consumer prices, the price of the capital good increases by the tax τ_k and the price to be paid for one unit of the consumption good increases by τ_c. Through these channels taxes directly affect individual's real wealth, however, it seems plausible that taxes do *not* directly affect the difficulty level.

[5] All references starting with characters refer to the web appendix of this paper available from the authors.

The size of the prototype is argued to increase in the amount of time and resources R_t spent on developing κ_t. Longer research could imply a larger prototype. We capture these aspects in a simple and tractable way by keeping κ_t proportional to the (tax-independent) size of total wealth an instant before a jump, $K_{t-}^{obs} = La_{t-}$,

$$\kappa_t \equiv \kappa \frac{1+\tau_c}{1+\tau_k} K_{t-}^{obs} = \kappa B^{-q} K_{t-}, \quad 0 < \kappa \ll 1. \tag{16}$$

While it may be debatable whether or not the payoffs of the risky research project, as a kind of income, could be subject to taxation, it seems a plausible assumption that the payoff itself, that is the size of the prototype, does not directly depend on tax rates.

3 Equilibrium dynamics

Solving the model requires conditions for optimal consumption and research expenditure. These two conditions, together with the capital accumulation constraint (8), market clearing, and optimality conditions of competitive firms provide a system consisting of six equations that determines the time paths of variables of interest K_t, C_t, R_t, Y_t, w_t and r_t.

This type of system can best be understood by introducing auxiliary variables: In the classical Solow growth model, capital per effective worker (or efficiency unit) is shown to converge to a non-stochastic steady state and transitional dynamics can be separated from the analysis of long-run growth. In the present context, we define $\hat{K}_t$ and $\hat{C}_t$ as

$$\hat{K}_t \equiv K_t/A^{q/\alpha} = B^{-q_t} K_t/A_t^q, \quad \hat{C}_t \equiv C_t/A^q, \tag{17}$$

which is almost identical to capital and consumption per effective worker as labor supply is constant here. These variables allow us to separate the analysis of cyclical properties of the model from long-run growth. In what follows, we denote $\hat{K}_t$ and $\hat{C}_t$ as 'cyclical components' of K_t and C_t since $A^{q/\alpha}$ and A^q turn out to be the stochastic trends for the capital index in units of vintage 0 and in units of the most recent vintage q, respectively. All variables expressed in units of the consumption good (including the capital stock in units of the most recent vintage) share the same trend, A^q, as from (17). Thus dividing non-stationary variables such as Y_t, C_t, R_t, I_t , w_t and G_t by the common stochastic trend A^q, these 'cyclical variables' turn out to be stationary and within a bounded range (r_t is stationary by construction).

3.1 An explicit solution

It would be interesting to analyze such a system in all generality. However, one would run the risk of losing the big picture and instead be overwhelmed by many small results. As the main objective of this paper is closed-form measures of volatility, we restrict ourselves to a particular parameter set of the model that allows very sharp analytical results.

Theorem 1 *If relative risk aversion equals the output elasticity of the capital stock, $\sigma = \alpha$, we obtain an equilibrium with optimal policy functions*

$$\hat{C}_t = \Psi \hat{K}_t, \quad \hat{R}_t = \Gamma \hat{K}_t, \tag{18}$$

where we define constants

$$\Psi \equiv \frac{1+\tau_k}{1+\tau_c}\left(\frac{1-\sigma}{\sigma}\left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right) + \frac{\rho + \lambda - (1-s)\lambda\xi^{-\sigma}}{\sigma} - \frac{1+\tau_r}{1+\tau_k}\lambda^{1-\gamma}D\right), \tag{19}$$

$$\Gamma \equiv \lambda^{1-\gamma}D, \tag{20}$$

$$\xi \equiv 1+\kappa-s, \tag{21}$$

and the arrival rate becomes

$$\lambda = \left(\frac{1+\tau_k}{1+\tau_r}\frac{\kappa}{D}\xi^{-\sigma}\right)^{\frac{1-\gamma}{\gamma}}. \tag{22}$$

Proof. see app. B.3 ■

Suppose the technological improvement (or economic depreciation s) of an innovation is sufficiently large relative to the size of the new prototype $\kappa \ll 1$ such that $\xi \leq 1$, or $\kappa \leq s$. Intuitively this assumption ensures that cyclical variables are accumulated and not reduced over the cycle which seems the only empirically plausible assumption (cf. Wälde, 2005). It follows from (13) and (16) that wealth, a_t/a_{t-}, and thus consumption, C_t/C_{t-} or research R_t/R_{t-} jump by the factor ξ or equivalently by $\kappa - s$ percent, whereas output Y_t/Y_{t-} from (6), (8), and (16) increases by $(1+B\kappa)^{\alpha}$ immediately after successful research.

The parametric restriction $\sigma = \alpha$ implies a relatively high intertemporal elasticity of substitution above unity (or risk aversion below unity). While there is supporting empirical evidence (as in Vissing-Jørgensen, 2002; Gruber, 2006), our fundamental insights about the presence of tax effects on volatility, as well as the channels through which taxes affect volatility, will not depend on this restriction. Further, it has proven very useful in the macro literature to study equilibrium dynamics (e.g. Chang, 1988; Xie, 1991, 1994; Boucekkine and Tamarit, 2004; Smith, 2007; Posch, 2009b).

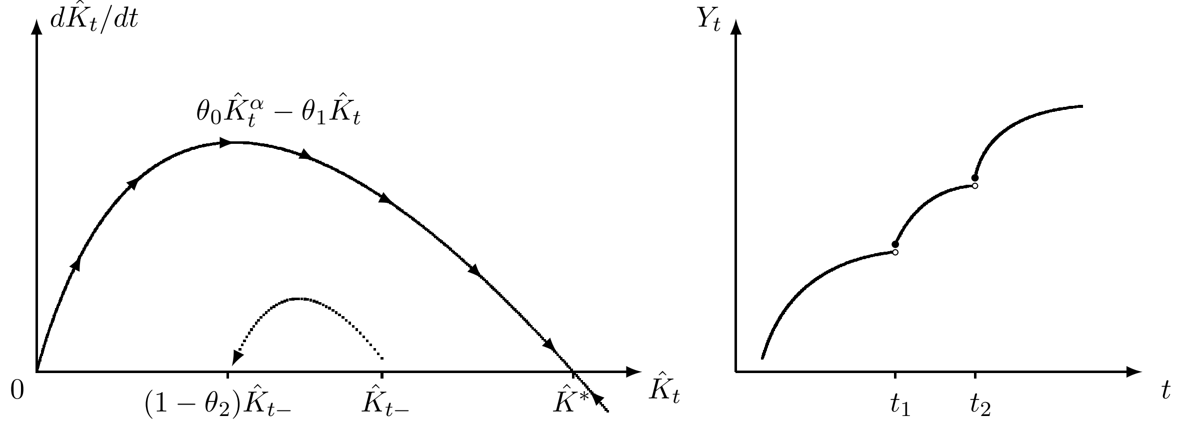
3.2 Cyclical growth

Exploiting the implications of Theorem 1, we can obtain the general-equilibrium behavior of agents in a way as simple as in the deterministic Solow growth model with a constant saving rate, even though we have forward-looking agents and an uncertain environment.

In terms of cyclical components, using Itô's formula (change of variables) together with capital accumulation in (8), the market clearing condition in (4) and the detrending rule (17), our capital index follows (cf. app. B.3)

$$d\hat{K}_t = \left(\hat{Y}_t - \hat{C}_t - \hat{R}_t - \hat{G}_t - \delta\hat{K}_t\right)dt + \left(A^{-1}\xi - 1\right)\hat{K}_{t-}dq_t.$$

Figure 1: Dynamics of cyclical capital and growth cycles



Note: This figure illustrates equilibrium dynamics of cyclical capital stock (intensive form) (left panel), and the resulting endogenous growth cycles for output (right panel), where jumps occur at t_1 and t_2, each starting a new growth cycle.

Inserting optimal consumption and research expenditure from (18) of Theorem 1, as well as government revenues using $\hat{G}_t = A^{-q}G_t$ and government revenues yields

$$\begin{aligned} d\hat{K}_t = &\left(\frac{1-\tau_i}{1+\tau_k}\hat{Y}_t - \left(\frac{1+\tau_c}{1+\tau_k}\Psi + \frac{1+\tau_r}{1+\tau_k}\Gamma + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)\hat{K}_t\right)dt + \left(A^{-1}\xi - 1\right)\hat{K}_{t-}dq_t \\ \equiv &\left(\theta_0\hat{K}_t^\alpha - \theta_1\hat{K}_t\right)dt - \theta_2\hat{K}_{t-}dq_t, \end{aligned} \tag{23}$$

where we inserted $\hat{Y}_t = A^{-q}Y_t = \hat{K}_t^\alpha L^{1-\alpha}$ from (6) and defined parameters

$$\theta_0 \equiv \frac{1-\tau_i}{1+\tau_k}L^{1-\alpha}, \quad \theta_1 \equiv \frac{1}{\sigma}\left(\rho + \lambda - (1-s)\,\lambda\xi^{-\sigma} + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right), \quad \theta_2 \equiv 1 - A^{-1}\xi.$$

As a result, similar to Solow's growth model our model implies a one-dimensional SDE with non-linear drift in (23), but satisfying utility-maximizing behavior of agents for $\alpha = \sigma$. Note that θ_1 is obtained when inserting Ψ and Γ from (19) and (20) respectively.

The terms in (23) containing parameters θ_0 through θ_2 have an economic interpretation: $\theta_0\hat{K}_t^\alpha$ represents cyclical output of this economy reduced by taxation, $\theta_1\hat{K}_t$ denotes effective resource allocation to research, private and government consumption, as well as physical depreciation. As from (23), the term $\theta_1 - \alpha\theta_0\hat{K}_t^{\alpha-1}$ denotes the speed of convergence towards the non-stochastic steady state, $\hat{K}^* = (\theta_0/\theta_1)^{\frac{1}{1-\alpha}}$. When an innovation occurs, the parameter θ_2 denotes the size of the jump in the cyclical capital index. For illustration, fig. 1 plots $\hat{K}_t$ against the deterministic part of cyclical capital (left panel). Note that the non-linear deterministic part in equation (23) implies that the speed of convergence (the slope in fig. 1) depends on the level of $\hat{K}_t$, thus changes as $\hat{K}_t$ moves towards $(1-\alpha)\theta_1$.

We can now start our analysis as in the neoclassical growth model. Suppose $\hat{K}_0$ is the initial capital stock, $0 < \hat{K}_0 < \hat{K}^*$.[6] Households optimally allocate their savings to either research or capital accumulation. Assuming a certain length of time without jumps, i.e., without successful innovation, the economy grows due to capital accumulation and converges to the non-stochastic steady state, $\hat{K}^*$. As in the Solow model, growth rates are initially high and approach zero. Once a jump occurs, $q_t = q_{t-} + 1$, the capital stock of the new vintage $q + 1$ increases by κ_t as in (2). This leads to a discrete increase of the capital index by the effective size, $B^{q+1}\kappa_t$. Although the capital stock increases by the size of the new prototype, our assumption about κ being sufficiently small ensures $\xi \leq 1$ in (21), and cyclical capital $\hat{K}_t$ unambiguously decreases because the frontier technology shifts outwards (cf. fig. 1). Due to higher marginal products, capital accumulation becomes more profitable, growth rates jump to a higher level approaching zero again until the next innovation occurs.

The discrete increases of labor productivity by A imply a step function in *vintage-specific* total factor productivity (TFP), in contrast to the smooth evolution in traditional balanced growth models à la Romer (1990). As a result, output in this economy is growing through cycles as illustrated in fig. 1 and fluctuations are a natural phenomenon in a growing economy. However, this step function of vintage-specific TFP does *not* imply that there are discrete jumps in aggregate TFP. As we show in (6), vintages of capital goods can easily be aggregated to an index (7) which weights them such that prices fully reflect differences in productivity and the *aggregate* TFP is constant and equal to unity.

4 Volatility measures

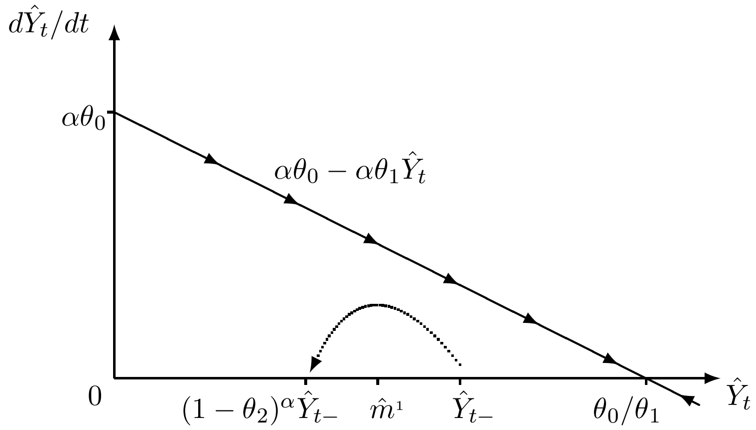
Volatility can be measured in many ways. The empirical literature focuses on either the standard deviation of detrended variables or output growth rates. In this study, we derive and employ three closed-form measures for volatility. Our first two measures are based on the cyclical component of output and the second one is based on output growth rates.

Cyclical component. There are many techniques to obtain stationary variables. Given their complexity, virtually none of these filters allow us to derive tractable cyclical variables which imply closed-form measures of volatility. Any deterministic filter would give no meaningful cyclical variables in our model as the second moment would not be bounded. We therefore use a very simple stochastic filter, a Solow-type detrending rule used in (17), to compute the cyclical component of output. It captures our stochastic trend by a step function A^q caused by the discrete increases of q_t. In fact, we decompose the output series Y_t into a stochastic trend A^q and a stationary cyclical component $\hat{Y}_t$. Using (6) and (17), they are related according to $\hat{Y}_t \equiv A^{-q} Y_t = \hat{K}_t^{\alpha} L^{1-\alpha}$.

In order to obtain an analytical measure now suppose that $\alpha = 0.5$ and normalize $L = 1$.

[6] Without loss of generality, we abstract from the case where $\hat{K}_0 > \hat{K}^*$. Given that $\theta_2 < 1$, at some point in time cyclical capital stock will be below its non-stochastic steady state with probability one.

Figure 2: The cyclical component of output



Note: This figure illustrates the dynamics of cyclical output with constant speed of convergence (the slope of $d\hat{Y}_t/dt$). Otherwise the dynamics are similar to those of cyclical capital stock (compare with fig. 1).

We restrict our focus to this parametric restriction for the following two reasons. First, we are able to compute all moments of cyclical output explicitly as they are given by the solution to an ordinary differential equation. Second, we can show that it is reasonable to assume that the qualitative effects of taxation on other empirically identifiable measures such as consumption are equivalent because the channels are the same.[7]

Using Itô's formula and (23), cyclical output follows (cf. app. 8.1 for a discussion)

$$\begin{aligned} d\hat{Y}_t &= \alpha\hat{K}_t^{\alpha-1}\big(\theta_0\hat{K}_t^\alpha - \theta_1\hat{K}_t\big)dt + \big((1-\theta_2)^\alpha\hat{K}_{t-}^\alpha L^{1-\alpha} - \hat{K}_{t-}^\alpha L^{1-\alpha}\big)dq_t \\ &= \big(\alpha\theta_0 - \alpha\theta_1\hat{Y}_t\big)dt - \big(1-(1-\theta_2)^\alpha\big)\hat{Y}_{t-}dq_t, \quad \text{where} \quad \alpha = 0.5,\ L = 1. \end{aligned} \tag{24}$$

In fact, the dynamics of (24) are similar to the evolution of cyclical capital (23). The speed of convergence, $\alpha\theta_1$ is constant (the slope in fig. 2). For our parametric restriction $\alpha = 0.5$ the SDE in (24) has a linear drift. Again, we can gain insights from plotting $\alpha\theta_0 - \alpha\theta_1\hat{Y}_t$ on the vertical axis, while $\hat{Y}_t$ is depicted on the horizontal axis (fig. 2). Obviously, cyclical output has support between 0 and its non-stochastic steady state, $0 < \hat{Y}_0 < \hat{Y}^*$, which from (24) is given by θ_0/θ_1. Starting from $\hat{Y}_0$, as long as no innovation takes place, the cyclical component approaches its upper bound. Each successful research project reduces cyclical output by $(1-(1-\theta_2)^\alpha)\hat{Y}_{t-}$, or $1-(1-\theta_2)^\alpha$ percent of cyclical output an instant before the innovation (which ensures that cyclical output always remains positive).

Exploiting the methods in García and Griego (1994), we can compute analytical moments of the cyclical component and a closed-form measure of volatility. We use the coefficient of

[7] A similar analysis could be undertaken for $\alpha = \sigma$. The analytical measure would then describe the volatility of instantaneous utility. As the empirical counterpart for utility is not as obvious as for output, we prefer to work with $\alpha = 0.5$.

variation (cv) as a scale-independent measure for volatility (see app. 8.3 for details),

$$cv(\hat{Y}_t)^2 \;\equiv\; \lim_{t\to\infty} \frac{Var_0(\hat{Y}_t)}{\left(E_0(\hat{Y}_t)\right)^2} = \frac{1-(1-\theta_2)^\alpha - \alpha\theta_2}{\alpha\theta_1/\lambda + \alpha\theta_2}, \quad \text{where} \quad \alpha = 0.5. \tag{25}$$

Looking at the cv shows that it is independent of θ_0. This is not surprising as θ_0 is a scaling parameter and the cv is scale-independent. This can intuitively be understood from fig. 2 where the effect of θ_0 on the cyclical component could be removed by scaling both axes with $1/\theta_0$. A lower speed of convergence, $\alpha\theta_1$, implies a higher measure of relative dispersion, cv. Clearly, the slower the economy approaches its non-stochastic steady state, the higher the overall variability of cyclical components is. The jump term θ_2 and the arrival rate, λ (note that θ_1/λ decreases in λ), have a positive effect on cv, meaning that larger and more frequent jumps imply a higher measure of relative dispersion.

We can also derive a measure of volatility based on growth rates of cyclical components of output, $\Delta\hat{y}_t \equiv \ln\hat{Y}_t - \ln\hat{Y}_{t-\Delta}$. Since this measure involves computing the variance of an integral over capital rewards, an analytical derivation – as for our measure based on the coefficient of variation – is not possible. Thus we use a deterministic Taylor expansion and neglect third-order terms in order to obtain (see app. 8.3 for a full derivation)

$$\begin{aligned} Var(\Delta\hat{y}_t) \;&\equiv\; \lim_{t\to\infty} Var_0\left(\frac{1-\tau_i}{1+\tau_k}\int_{t-\Delta}^{t} r_s ds\right) + (\alpha\ln(1-\theta_2))^2\lambda\Delta \\ &\approx\; \lim_{t\to\infty} Var_0\left(\frac{1-\tau_i}{1+\tau_k} r_t\right)\Delta^2 + (\alpha\ln(1-\theta_2))^2\lambda\Delta \\ &=\; \tfrac{\alpha^2}{1-\alpha}\left(\tfrac{1-(1-\theta_2)^{1-\alpha}}{(1-\theta_2)^{1-\alpha}} + (1-\alpha)\ln(1-\theta_2)\right)(\theta_1 - \ln(1-\theta_2)\lambda)\,\lambda\Delta^2 \\ &\quad\; + (\alpha\ln(1-\theta_2))^2\,\lambda\Delta. \end{aligned} \tag{26}$$

Obviously, this measure shares the property of scale independence with the cv because we consider growth rates, which by construction are scale independent. It can be interpreted as an approximation for the variance of growth rates of cyclical output.

Output growth rates. An empirically more obvious measure is based on output growth rates, $\Delta y_t \equiv \ln Y_t - \ln Y_{t-\Delta}$. Using the detrending rule (17) it can be shown that

$$\begin{aligned} Var(\Delta y_t) \;&\approx\; \tfrac{\alpha^2}{1-\alpha}\left(\tfrac{1-(1-\theta_2)^{1-\alpha}}{(1-\theta_2)^{1-\alpha}} + (1-\alpha)\ln(1-\theta_2)\right)(\theta_1 - \ln(1-\theta_2)\lambda)\,\lambda\Delta^2 \\ &\quad\; + (\alpha\ln(1-\theta_2) + \ln A)^2\,\lambda\Delta, \end{aligned} \tag{27}$$

which again is only an approximation since we neglect third-order terms.

We can also derive an explicit expression for mean growth,

$$E(\Delta y_t) \equiv \lim_{t\to\infty} E_0(\Delta y_t) \;=\; E(\Delta\hat{y}_t) + E(\Delta q_t)\ln A \;=\; \lambda\ln A\Delta. \tag{28}$$

This long-run expected growth rate of the common stochastic trend is determined by the arrival rate of new technologies. From (22), λ increases in the investment tax, τ_k, and decreases in the tax on research, τ_r. We study the effects of taxes on volatility below.

Note that this measure comes closest to output growth residuals used in Ramey-Ramey type regressions. As our model does not contain exogenous disturbances, a regression analysis of data produced by a simulated version of our model would not have to control for any disturbances. In other words, Δy_t in our theoretical model corresponds to the output growth residuals in the Ramey-Ramey regression. The next section will analyze, inter alia, how taxes affect our theoretical 'output growth residual' Δy_t.

5 Volatility and taxation

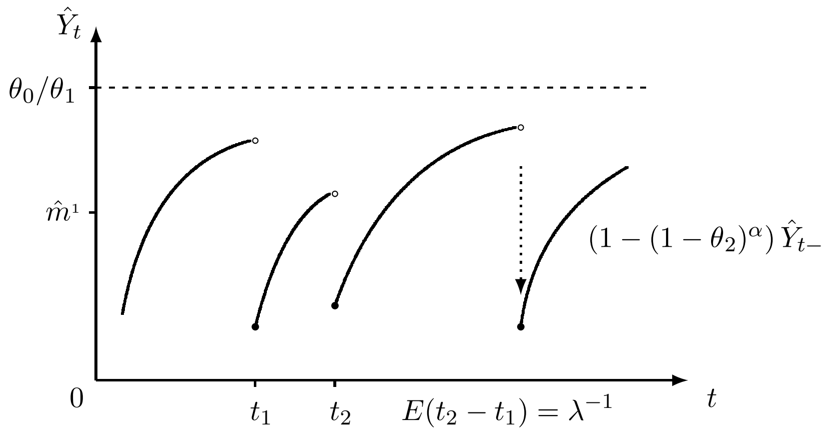
5.1 Theoretical findings

Our measure of volatility in (25) is affected through three channels, the speed of convergence $\alpha\theta_1$, the jump size $1-(1-\theta_2)^\alpha$, and the arrival rate λ. As shown, these determinants appear in the measures based on growth rates in (26) and (27). For illustration, the interpretation of these channels is based on cyclical output in (24). Consider an arbitrary realization of the cyclical component in fig. 3. In line with our previous results, the speed of convergence, $\alpha\theta_1$, determines the range of cyclical output $(0, \theta_0/\theta_1)$. The upper limit corresponds to the non-stochastic steady state for cyclical capital, $\hat{K}^* = (\theta_0/\theta_1)^{\frac{1}{1-\alpha}}$. However, the only parameter which is relevant for the relative dispersion of cyclical output is θ_1 (θ_0 is a scaling parameter). From its definition in (24) and the discussion of (23), it is clear that $\alpha\theta_1$ denotes the effective resource allocation to both research expenditures and total consumption. The arrival rate λ measures the frequency of jumps (the inverse measures the expected length of growth cycles). Finally, the size of the jump is measured by $1-(1-\theta_2)^\alpha$. Hence we find that output volatility depends on the *level* of taxes if at least one of the following three channels, (i) the speed of convergence, (ii) the jump size or (iii) the arrival rate depends on taxes.[8]

To understand the effects of taxation on macro volatility, we may restrict attention to the speed of convergence and the arrival rate (or jump probability), because the jump size does not depend on taxes. The independence of θ_2 follows from the fact that the jump in consumption, $\xi = 1 - s + \kappa$, from (24) is not affected by taxes. Economically, this result is obtained because payoffs κ are not taxed and economic depreciation, s, does not imply tax-exemption as does physical depreciation, δ. The tax effects on the arrival rate λ are obtained from (22). The parameter θ_1 in (24) depends on taxes both directly and indirectly through the arrival rate. The direct effect reflects the effective rate of physical depreciation, $\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a$, whereas the indirect effect reflects tax effects on the arrival rate, λ, which in turn are due to changes in private consumption, $\hat{C}_t$, research expenditures, $\hat{R}_t$, and government consumption, $\hat{G}_t$. Inserting λ into θ_1 gives unambiguous results (cf. app. D.1 for details).

[8] If growth and cycles are exogenous, i.e., if there is an exogenous arrival rate λ without research, the model describes a continuous-time RBC model with vintage-specific capital. In this case, macro volatility is partly endogenous and affected by taxation through the speed of convergence, $\alpha\theta_1$.

Figure 3: The cyclical component of output and its determinants



Note: This figure illustrates the determinants of the cyclical component of output and thus the coefficient of variation (cv) using an arbitrary realization of the SDE in (24). Our measure in (25) is determined by the speed of convergence, $\alpha\theta_1$, the arrival rate, λ, and the jump size, $1-(1-\theta_2)^\alpha$.

For reading convenience, the qualitative results are summarized in tab. 1.

5.2 Comparative statics

Let us now combine the effects of our three channels on output volatility measured by (25) in a comparative static analysis. As we have only two tax-dependent channels, the speed of convergence, $\alpha\theta_1$, and the arrival rate, λ, taxes affect the variance of the limiting distribution of stationary output by either changing the speed of convergence (without affecting λ in θ_1), the arrival rate, or both. Clearly, a tax which has no effect on θ_1 and λ, does not affect our measures either. The tax on consumption expenditures, τ_c, is such a tax because government consumption completely offsets changes in private consumption.

When taxing wealth, τ_a, the arrival rate λ is not affected. The speed of convergence, $\alpha\theta_1$, increases which causes cv in (25) to decline. Economically, τ_a decreases the households' return on savings, or equivalently, increases the effective rate of depreciation. This in turn implies a lower non-stochastic steady-state, $\hat{K}^*$, and more resources are used for consumption and research. Holding constant the length of a cycle but 'squeezing' cyclical output in fig. 3, the relative dispersion of cyclical output must be lower.

An increase in the income tax, τ_i, reduces the speed of convergence $\alpha\theta_1$ but does not affect the jump probability, λ. As a consequence, volatility unambiguously increases in this tax. How can this result be understood? The parameter θ_1 in (24) decreases for the following reason: only *net* investment is taxed (as discussed above). This means that a higher tax on income increases the positive effect of the refunding policy and reduces the impact of the depreciation rate, δ. A lower effective depreciation rate increases incentives for capital

Table 1: Qualitative effects of taxes on composite parameters, volatility and growth

		Taxes				
		τ_i (income)	τ_c (consumption)	τ_r (research)	τ_k (investment)	τ_a (wealth)
$\alpha\theta_1$	(speed of convergence)	$-$	0	$-$	$+^\dagger$	$+$
$1-(1-\theta_2)^\alpha$	(jump size)	0	0	0	0	0
λ	(arrival rate)	0	0	$-$	$+$	0
$E(\Delta y_t)$	(mean growth rate)	0	0	$-$	$+$	0
$Var(\Delta y_t)$	(variance of growth rates)	$-$	0	$-$	$+^\dagger$	$+$
$cv(\hat{Y}_t)$	(coefficient of variation)	$+$	0	$-$	$+$	$-$

† for δ sufficiently small

Note: This table shows the qualitative tax effects of the *level* of tax rates on output volatility and growth and their components. Our measures include the speed of convergence of cyclical output, $\alpha\theta_1$, the jump size, $1-(1-\theta_2)^\alpha$, and the arrival rate, λ, Δy_t, and the coefficient of variation of cyclical output, $cv(\hat{Y}_t)$.

accumulation, and the non-stochastic steady-state capital stock, $\hat{K}^*$, increases.

For the taxes on research, τ_r, and investment, τ_k, the results are less clear-cut. With these taxes the arrival rate λ is affected which in turn changes cv directly and indirectly through θ_1. The direct effect of λ on cv is unambiguously positive. Computing the derivatives, however, we obtain the results for our measures of volatility as in tab. 1. A higher tax on research depresses the arrival rate and the ratio θ_1/λ increases, which in turn decreases cv in (25). Intuitively, higher rates τ_r make investment in research less profitable and the arrival rate falls. Less frequent jumps imply a lower relative dispersion of cyclical output. A lower λ also decreases θ_1, thus less resources are used for consumption and research. This implies a larger range $1/\theta_1$ in fig. 2 and higher volatility. The indirect effect through the lower speed of convergence does not compensate the direct effect of a lower arrival rate. Hence, the ratio θ_1/λ increases and cv in (25) decreases. A similar interpretation can be given for τ_k.

Given the discussion above, we can now understand why measures based on output growth rates may also depend on taxes. Consider the speed of convergence $\alpha\theta_1$. As shown above, an increase in θ_1 decreases the range of the cyclical component. Obviously, this decreases the cv and variables in efficiency units, but increases the variance of capital rewards. This in turn implies a higher variance of output growth rates. Hence, the tax effects implied through the propagation of shocks is reversed for measures based on growth rates. However, the qualitative effects on the arrival rate are identical to our measures of relative dispersion.

5.3 The link between volatility and growth

We are now prepared to make our main point. For a given tax policy, our economy follows a certain cyclical growth path. Now imagine a second economy with a different tax policy and a third one with yet another tax policy and so on. Given our comparative static results, it is straightforward to understand why growth and volatility are correlated and that this correlation can take any sign – depending on cross-country differences in tax *levels* and which measure we use for volatility.

Suppose two countries differ only in the *level* of the investment tax (value added tax on physical investment goods). Tab. 1 shows that both growth and volatility increase in the investment tax (independently of which measure is used), as resources are shifted to R&D. In a cross-section of countries, we expect a positive correlation between volatility and growth. The same positive correlation would exist if countries differ only in their tax on research. Our model predicts a negative correlation between volatility and growth for various combinations of tax rates. An example of this is when countries with a high tax on investment also have a high tax on income. The investment tax increases volatility and growth, the income tax decreases volatility (focusing on the measure based on growth rates in tab. 1). If the negative effect is stronger than the volatility-increasing effect of the investment tax, we expect a negative correlation between investment and growth.

The reader may want to discuss our examples on tax policy structures as well as the generality of our parametric restrictions $\alpha = \sigma$ (the sign of tax effects may be ambiguous in the general case). Yet, our general point remains: differences in economic policies across countries may imply differences in output growth rates and output volatility. Depending on cross-country differences, correlations of any magnitude and sign can occur.

6 Empirical implications and findings

6.1 Implications for empirical research

Given our theoretical findings on the effects of tax *levels* on both volatility and growth, our message for empirical work is that a volatility-growth regression in the spirit of Ramey and Ramey (1995) should include controls of this type. To elaborate on this point, consider the following extension of Ramey and Ramey,

$$\Delta y_{it} = \nu\sigma_{it} + \theta X_{it} + \varepsilon_{it}, \quad \text{where} \quad \varepsilon_{it} \sim N(0, \sigma_{it}^2), \tag{29a}$$

$$\log(\sigma_{it}^2) = \alpha_i + \mu_t + \beta Z_{it}. \tag{29b}$$

Δy_{it} is the growth rate of output for country i in year t computed as log difference; σ_{it} is the standard deviation of residuals; X_{it} is a vector of control variables such as the Levine-Renelt variables and/or taxes; Z_{it} is a vector of control variables (could be a subset of X_{it}); α_i and λ_t are country and time fixed effects; θ and β are vectors of coefficients. The key parameter of interest in a volatility-growth analysis is ν, which links growth to volatility.

For $Z_{it} = 0$, this specification coincides with the basic Ramey-Ramey setup.[9] Given our

[9] The authors extend their basic framework where government-spending volatility replace the fixed-effects in the conditional variance equation. Proceeding in this direction does not change our results (not shown).

theoretical arguments, the conditional variance equation needs to include additional controls Z_{it}. As we have seen that the level of taxes can have an effect on volatility, we would expect β to be significant if Z_{it} measures the level of taxes. If those measures were constant over time, $Z_{it} = Z_i$, our proposed extension would be equivalent to country-specific fixed effects – as already included in the basic Ramey-Ramey setup. New empirical insights from our arguments therefore require sufficient variation in taxes over time. It is well-known from Mendoza-Razin-Tesar tax rates that such variation is present in the data.

We admit that though it illustrates our argument, the absence of uncertainty other than endogenous innovations makes it difficult to relate our theoretical measure of output volatility in (27) to the conditional variance of residuals (29b). In a stylized Ramey-Ramey framework, however, a regression of growth rates on a constant, $\Delta y_{it} = \mu + \varepsilon_{it}$ in which μ is the mean output growth rate shows that the conditional residual variance, by construction, coincides with our theoretical measure. This suggests that the variance of output growth rates and the variance of the error term may indeed share some important properties. We therefore feel justified in letting our empirical approach be guided by our theoretical model.[10]

6.2 Empirical findings

This section illustrates our message for empirical work. Our empirical sample consists of 20 countries for which we have data for the period from 1970 to 2009.[11] To measure the tax burden of a representative household on the macro level we follow Mendoza, Razin and Tesar (1994). We employ data from the OECD Revenue Statistics 2010 (OECD-iLibrary, http://dx.doi.org/10.1787/data-00262-en) and OECD Aggregate National Accounts 2010 (http://dx.doi.org/10.1787/na-ana-data-en) to obtain updated tax ratios and annual growth rates of real GDP per capita respectively (cf. Mendoza et al., 1994; Posch, 2009a). We also compute the relative price of investment from the Penn World Table 6.3 as in Restuccia and Urrutia (2001). This variable measures barriers to capital accumulation and growth. It turned out to be to be important for accounting for cross-country differences in investment rates.[12]

In tab. 2 we present the results from a joint estimation of equations (29a) and (29b). The first column is a specification with $Z_{it} = 0$, i.e., with no variables in the conditional variance equation apart from fixed effects. The only explanatory variable X_{it} in the growth equation is the price of aggregate investment over consumption ($RPRICE$). This specification extends the basic Ramey-Ramey setup by allowing for variations in the relative price of investment.

[10] Empirical work by Posch (2009a) has shown that the impact of taxes on volatility is very similar for a specification as in (29b) and a specification where the variance of the error term is replaced by observed output variances (estimated e.g. through rolling windows).

[11] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Korea, Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, United Kingdom, United States.

[12] We would like to thank a referee for drawing our attention to this work.

Table 2: The link between volatility and growth

OECD		Ramey-Ramey	Our claim
$LABOR_{i,t-1}$	θ_1		-0.15 (0.08) $\cdot$
$CAPITAL_{i,t-1}$	θ_2		0.06 (0.07)
$CONS_{i,t-1}$	θ_3		0.12 (0.08)
$CORP_{i,t-1}$	θ_4		-0.02 (0.05)
$RPRICE_{i,t-1}$	θ_5	-0.06 (0.01) ***	-0.07 (0.01) ***
$LABOR_{i,t-1}$	β_1		-8.84 (2.37) ***
$CAPITAL_{i,t-1}$	β_2		8.55 (2.22) ***
$CONS_{i,t-1}$	β_3		4.81 (3.17)
$CORP_{i,t-1}$	β_4		-4.19 (1.70) *
$\sigma_{i,t}$	ν	-1.61 (0.51) **	-2.46 (0.83) **
Degrees of freedom		672	664
Log-likelihood		-1894.8	-1918.1
Country fixed effects	θ_i	yes	yes
Country fixed effects	α_i	yes	yes
Time fixed effects	μ_t	yes	yes

Signif. codes: '***' 0.001 '**' 0.01 '*' 0.05 '·' 0.1

Notes: This table reports the semi-elasticities on output growth innovations treating variances as parameters using MLE. Similar to Ramey and Ramey (1995) all specifications include two lags of log real GDP per capita and a trend variable in the growth equation. Standard errors in parentheses. Including government spending volatility does not change our results, whereas its estimated parameter is not different from zero for any conventional significance level (not shown).

The column 'Ramey-Ramey' contains our benchmark results for understanding the effects of adding further controls to the conditional variance equation.

The column 'our claim' adds control variables to both the growth equation (29a) and the conditional variance equation (29b). We include four different taxes: the tax induced cost of dependent labor ($LABOR$), the tax induced cost of capital ($CAPITAL$), the consumption tax ($CONS$) and the corporate income tax ($CORP$). We find that 3 out of 4 tax measures are significant at the 5% level. Hence, the *level* of taxes indeed appears to affect volatility. For example, if $LABOR$ is increased by one percentage point, the variance of output growth rates decreases by 8.85 percent. We also find that the conventional Ramey-Ramey coefficient ν is affected sizably. Accounting for taxes in the conditional residual variance equation strengthens the previous findings of a statistically significant negative correlation between growth and volatility. Compared to our benchmark specification, the link remains negative and increases in absolute terms. It appears that previous estimates have been much too modest and the (negative) correlation is stronger than found so far.

As one caveat, it is well-known that although the Mendoza-Razin-Tesar tax rates are very useful on the one hand, it is difficult to link them closely to theoretical (marginal) tax rates on the other. For example, the income tax in our model is a composite of $LABOR$ and $CAPITAL$ in the estimation. This makes it difficult to conclude whether the empirical finding of a negative effect ($LABOR$) and a positive effect ($CAPITAL$) confirms or contradicts our theoretical prediction in tab. 1 that an income tax reduces volatility.

We conclude this short empirical application by stressing that our fundamental point made in the theoretical model – a *level* variable affects volatility and growth – is confirmed. Future work should construct theoretical models which closely replicate the construction of the empirical tax ratios. With such one-to-one correspondence between theoretical and empirical taxes, more insights can be gained into the volatility effect of taxes.

7 Conclusion

There is a growing literature studying the link between volatility and growth. This paper theoretically shows that the correlation between volatility and growth can be positive or negative. The sign of the correlation is determined inter alia by the economic policy implemented by the country under consideration.

We illustrate our main point by identifying taxes as the truly fundamental parameters which determine the growth rate and the degree of volatility of a country. Our theoretical contribution is that our measures of volatility are obtained analytically. This allows us to follow an analytical approach in understanding the channels through which the *level* of taxes affects both the volatility and the growth processes. We find that taxes determine the sign of the correlation between volatility and growth. For example, if taxes on wealth are used to facilitate R&D investment, growth and volatility are positively correlated. In contrast, if taxes on wealth are used to promote physical capital investment, a negative link may occur.

In an empirical application of the Ramey-Ramey type, we add levels of taxes to the growth and the conditional variance equation. We find that the growth-volatility link is much more negative than in regressions where taxes are not taken into account. In addition, taxes do indeed appear to affect volatility.

This paper opens up interesting future research avenues. Our theoretical model does not allow for any exogenous source of volatility. In an extended framework with both endogenous and exogenous shocks, one can identify which share of volatility is due to endogenous sources. This would provide a framework that allows to split the growth-volatility link into a correlation-component and into a causation-component.

8 Appendix

8.1 Cyclical components

Using (23) and Itô's formula, $d\hat{Y}_t = L^{1-\alpha} d\hat{K}_t^\alpha$, we compute

$$d\hat{K}_t^\alpha = \alpha\big(\theta_0 \hat{K}_t^{\alpha-1} - \theta_1\big)\hat{K}_t^\alpha dt - \left(1 - (1-\theta_2)^\alpha\right)\hat{K}_{t-}^\alpha dq_t,$$

which is an SDE with a non-linear drift component. The non-stochastic steady state is $\hat{K}^{\alpha *} = (\theta_0/\theta_1)^{\frac{\alpha}{1-\alpha}}$, the speed of convergence is $\alpha\theta_1 - (2\alpha - 1)\theta_0 \hat{K}_t^{\alpha-1}$ is *not* constant unless

$\alpha = 0.5$, and the jump term $1-(1-\theta_2)^{\alpha}$, increases (decreases) relative to $\hat{K}_t^{1-\sigma}$ for $\alpha > 0.5$ ($\alpha < 0.5$) and is the same for $\alpha = 0.5$.

8.2 Properties of the Poisson process

We use the martingale property of various expressions. These expressions are special cases of $\int_0^t f(q_s, s)\, dq_s - \lambda \int_0^t f(q_s, s)\, ds$, which is a martingale (cf. García and Griego, 1994), i.e.,

$$E_0\left(\int_0^t f(q_s, s)\, dq_s - \lambda \int_0^t f(q_s, s)\, ds\right) = 0, \tag{30}$$

where λ is the (constant) arrival rate of q_t.

8.3 Appendix for (25) and (26)

Use the integral version of (24), $\hat{Y}_t = \hat{Y}_0 + \int_0^t \left(\alpha\theta_0 - \alpha\theta_1 \hat{Y}_s\right) ds - \int_0^{t-} \left(1-(1-\theta_2)^{\alpha}\right)\hat{Y}_s dq_s$, and the martingale property (cf. app. 8.2), we obtain

$$E_0(\hat{Y}_t) = \hat{Y}_0 + \int_0^t \left(\alpha\theta_0 - \alpha\theta_1 E_0(\hat{Y}_s)\right) ds - \lambda \int_0^{t-} \left(1-(1-\theta_2)^{\alpha}\right) E_0(\hat{Y}_s) ds, \tag{31}$$

which gives the evolution of the first moment of $\hat{Y}_t$ as a linear ordinary differential equation (ODE) which can be solved and is shown to converge to a constant. Using a similar approach, higher-order moments can be computed easily.[13] In fact, denoting the nth moment by

$$\hat{m}_t^n \equiv E_0(\hat{Y}_t^n), \tag{32}$$

the first and second moment of the stationary distribution are given by

$$\hat{m}^1 \equiv \lim_{t\to\infty} \hat{m}_t^1 = \frac{\alpha\theta_0}{\alpha\theta_1 + \lambda(1-(1-\theta_2)^{\alpha})}, \tag{33}$$

$$\hat{m}^2 \equiv \lim_{t\to\infty} \hat{m}_t^2 = \frac{\theta_0}{\theta_1 + \lambda\theta_2} \hat{m}^1. \tag{34}$$

A scale-independent measure is the coefficient of variation (cv). Given that the variance of a random variable is the difference between its second moment and the square of its mean, it is defined by

$$cv^2 \equiv \lim_{t\to\infty} \frac{Var_0(\hat{Y}_t)}{\left(E_0(\hat{Y}_t)\right)^2} = \frac{1-(1-\theta_2)^{\alpha} - \alpha\theta_2}{\alpha\theta_1/\lambda + \alpha\theta_2}, \tag{35}$$

where for the second equality we inserted the moments from (33) and (34), respectively.

[13] The structure of the moments is remarkable as it shows that the distribution of $\hat{Y}_t$ exists, is unique and represents a generalization of the β-distribution (thanks to Christian Kleiber for pointing this out). As shown in Appendix 8.4, fairly complex expressions appear for state dependent moments.

In order to obtain moments based on growth rates, we use integral equations for the log-variables and exploit the martingale property (cf. app. 8.2). For cyclical capital it reads

$$\begin{aligned} d\ln\hat{K}_t &= \left(\theta_0\hat{K}_t^{\alpha-1}-\theta_1\right)dt+\left(\ln\hat{K}_t-\ln\hat{K}_{t-}\right)dq_t \\ &= \left(\theta_0\hat{K}_t^{\alpha-1}-\theta_1\right)dt+\ln(1-\theta_2)dq_t. \end{aligned} \tag{36}$$

Integrating gives the growth rate of cyclical capital per unit of time Δ as

$$\ln\hat{K}_t-\ln\hat{K}_{t-\Delta}=\int_{t-\Delta}^{t}\frac{1-\tau_i}{1+\tau_k}r_s/\alpha ds-\theta_1\Delta+\ln(1-\theta_2)(q_t-q_{t-\Delta}), \tag{37}$$

where we relate growth rates to the integrated process of capital rewards, $r_t=\alpha\hat{K}_t^{\alpha-1}L^{1-\alpha}$. Similarly, the growth rate of cyclical output is $\Delta\hat{y}_t\equiv\ln\hat{Y}_t-\ln\hat{Y}_{t-\Delta}=\alpha(\ln\hat{K}_t-\ln\hat{K}_{t-\Delta})$.[14] In order to calculate the variance of growth rates the following lemma is very useful.

Lemma 1 *Suppose that* $\ln\hat{K}_t$ *follows (36), then*

$$\lim_{t\to\infty}Cov_0\left(\ln\hat{K}_t-\ln\hat{K}_{t-\Delta},q_t-q_{t-\Delta}\right)=\ln(1-\theta_2)\lambda\Delta.$$

Proof. app. C.3 ■

After some algebra, we obtain the asymptotic variance as (cf. app. 8.7)

$$\lim_{t\to\infty}Var_0(\Delta\hat{y}_t)=\lim_{t\to\infty}Var_0\left(\frac{1-\tau_i}{1+\tau_k}\int_{t-\Delta}^{t}r_s ds\right)+(\alpha\ln(1-\theta_2))^2\lambda\Delta. \tag{38}$$

This result is remarkable because it shows that the variance of growth rates depends on the variance of the (integrated) process of capital rewards, which in turn follows

$$dr_t=c_1r_t(c_2-r_t)dt+c_3r_{t-}dq_t, \tag{39}$$

where $c_1\equiv\frac{1-\alpha}{\alpha}\frac{1-\tau_i}{1+\tau_k}$, $c_2\equiv(1-\alpha)\theta_1/c_1$, and $c_3\equiv(1-(1-\theta_2)^{1-\alpha})/(1-\theta_2)^{1-\alpha}$. In fact, this SDE describes the (transitional) equilibrium dynamics of capital rewards, often referred to as the stochastic Verhulst equation. It is shown that r has a unique limiting distribution, and the moments of the limiting distribution are available in closed-form (cf. app. 8.6)

$$E(r)=\frac{c_1c_2+\ln(1+c_3)\lambda}{c_1},\quad Var(r)=\frac{c_3\lambda-\ln(1+c_3)\lambda}{c_1}E(r). \tag{40}$$

We use the deterministic Taylor expansion to approximate the asymptotic variance of the integrated process by (neglecting third order terms, cf. Posch, 2009a)[15]

$$\lim_{t\to\infty}Var_0\left(\int_{t-\Delta}^{t}r_s ds\right)\approx\lim_{t\to\infty}Var_0\left(r_t\Delta\right)=Var(r)\Delta^2. \tag{41}$$

[14] Obviously the expected growth rate of cyclical variables per unit of time is zero. This result is intuitive because $\hat{K}_t$ is bounded between 0 and $\hat{K}$, which implies a stationary distribution (as illustrated in fig. 1).

[15] A precise measure would take into account the auto-covariance function based on asymptotic moments $\lim_{t\to\infty}E_0(r_sr_u)$ which turn out to be negligible in simulations. Joint moments depend on higher-order moments and because of the non-linear dynamics in (39) are not available analytically.

Starting from (38) and inserting (41),

$$Var(\Delta\hat{y}_t) \equiv \lim_{t\to\infty} Var_0(\Delta\hat{y}_t) \approx \lim_{t\to\infty} Var_0\left(\frac{1-\tau_i}{1+\tau_k} r_t\right)\Delta^2 + (\alpha\ln(1-\theta_2))^2\lambda\Delta,$$

which is (26) in the main text.

8.4 Computing moments

Express (31) as a differential equation and use (32) to obtain $d\hat{m}_t^1 = (\alpha\theta_0 - (\alpha\theta_1 + \lambda\vartheta_2)\hat{m}_t^1)\,dt$, where $\vartheta_2 \equiv 1-(1-\theta_2)^\alpha$. Hence, the first moment follows a linear ODE with solution

$$\hat{m}_t^1 = e^{-(\alpha\theta_1+\lambda\vartheta_2)t}\left(\hat{m}_0^1 + \int_0^t e^{(\alpha\theta_1+\lambda\vartheta_2)s}\alpha\theta_0 ds\right) = e^{-(\alpha\theta_1+\lambda\vartheta_2)t}\left(\hat{m}_0^1 + \alpha\theta_0\frac{e^{(\alpha\theta_1+\lambda\vartheta_2)t}-1}{\alpha\theta_1+\lambda\vartheta_2}\right).$$

It can be simplified to

$$\hat{m}_t^1 = e^{-(\alpha\theta_1+\lambda\vartheta_2)t}\left(\hat{m}_0^1 - \frac{\alpha\theta_0}{\alpha\theta_1+\lambda\vartheta_2}\right) + \frac{\alpha\theta_0}{\alpha\theta_1+\lambda\vartheta_2}. \tag{42}$$

As $\alpha\theta_1 + \lambda\vartheta_2 > 0$, the first moment of $\hat{Y}_t$ is in the long run given by (33). Similarly, for higher moments, the basic ODE determining the evolution of $\hat{Y}_t^n$ is from (24)

$$\begin{aligned} d\hat{Y}_t^n &= n\hat{Y}_t^{n-1}\big(\alpha\theta_0 - \alpha\theta_1\hat{Y}_t\big)dt - (1-(1-\theta_2)^{\alpha n})\,\hat{Y}_{t-}^n dq_t \\ &= n\big(\alpha\theta_0\hat{Y}_t^{n-1} - \alpha\theta_1\hat{Y}_t^n\big)dt - (1-(1-\theta_2)^{\alpha n})\,\hat{Y}_{t-}^n dq_t. \end{aligned} \tag{43}$$

Using the integral version, applying expectations and the martingale result (30), we obtain $dE_0\hat{Y}_t^n = \big(n\alpha\theta_0 E_0\hat{Y}_t^{n-1} - (n\alpha\theta_1 + \lambda(1-(1-\theta_2)^{\alpha n}))\,E_0\hat{Y}_t^n\big)dt$. Using (32) we get

$$d\hat{m}_t^n = (n\alpha\theta_0\hat{m}_t^{n-1} - (n\alpha\theta_1 + \lambda(1-(1-\theta_2)^{\alpha n}))\,\hat{m}_t^n)\,dt. \tag{44}$$

It shows that all moments converge to finite limits for $t\to\infty$. For the first moment, this follows from (42). The proofs for higher moments follow an identical approach. In short, for asymptotic moments where $d\hat{m}_t^n/dt = 0$, we obtain from (44)

$$\hat{m}^n = \frac{n\alpha\theta_0}{n\alpha\theta_1 + \lambda(1-(1-\theta_2)^{\alpha n})}\hat{m}^{n-1}. \tag{45}$$

Thus $n=2$ implies (34), with $n=1$ it becomes (33), and by definition $\hat{m}^0 = 1$.

8.5 Limiting distribution

If the nth moment $\hat{m}_t^n \equiv E_0(\hat{Y}^n)$ has bounded support, then $\hat{m}^j \equiv \lim_{t\to\infty} E_0(\hat{Y}_t^j)$ is the jth moment of the limiting distribution for any $j < n$, and the moments in (33) and (34) converge to the moments of the limiting distribution. Moreover, $\hat{Y}_t$ has a unique limiting distribution (Rao, 1973, p.121; Casella and Berger, 2001, Theorem 2.3.11.). In other words, the sequence $\{\hat{Y}_t\}_{t=t_0}^{\infty}$ converges in distribution to a random variable $\hat{Y}$,

$$\hat{Y}_t \overset{\mathcal{D}}{\to} \hat{Y} \quad \text{where} \quad 0 < \hat{Y}_t < \hat{Y}^*. \tag{46}$$

In fact, the limiting density of any smooth transformation of $\hat{Y}_t$ is determined by the change of variable formula for densities (cf. Merton, 1975).

By inspection of moments in (45), $\hat{Y}$ has a generalized β-distribution. For $\theta_2 = 1$, the moments in (45) are $\hat{m}^n = \frac{n\alpha\theta_0}{n\alpha\theta_1+\lambda}\hat{m}^{n-1}$. Starting from $\hat{m}^0 = 1$, repeated inserting yields

$$\hat{m}^n = \frac{(\alpha\theta_0)^n n!}{\prod_{i=1}^n (i\alpha\theta_1 + \lambda)} = \left(\frac{\theta_0}{\theta_1}\right)^n \frac{\Gamma(n+1)}{\prod_{i=1}^n (i + \lambda/(\alpha\theta_1))} = \left(\frac{\theta_0}{\theta_1}\right)^n \frac{\Gamma(n+1)\Gamma(1+\lambda/(\alpha\theta_1))}{\Gamma(n+1+\lambda/(\alpha\theta_1))},$$

where Γ is the gamma function. Apart from the scaling factor $(\theta_0/\theta_1)^n$, the last expression denotes the nth moment of a β-distribution with parameters 1 and $\lambda/(\alpha\theta_1)$. Hence, $\hat{Y}$ has the asymptotic representation $\hat{Y} = (\theta_0/\theta_1)^n X$, where $X \sim \text{Beta}(1, \lambda/(\alpha\theta_1))$. For $\theta_2 \neq 1$, we obtain a generalized β-distribution which, to the best of our knowledge, has not been encountered before. Analyzing its properties in detail should be done in future research.

8.6 Moments of the rental rate of capital

Along the lines of our derivations for (46) it can be shown that r_t is a smooth transformation of a cyclical variable for the more general case $\alpha = \sigma$. Hence, the sequence $\{r_t\}_{t=t_0}^{\infty}$ converges in distribution to a random variable r (compare also to Posch, 2009b),

$$r_t \overset{\mathcal{D}}{\to} r \quad \text{where} \quad r^* < r_t < \infty. \tag{47}$$

Using (23) and $r_t = B^q \alpha K_t^{\alpha-1} L^{1-\alpha} = \alpha \hat{K}_t^{\alpha-1} L^{1-\alpha}$ we obtain

$$\begin{aligned} d\hat{K}_t^{\alpha-1} &= (\alpha-1)K_t^{\alpha-2}\big(\theta_0 \hat{K}_t^{\alpha} - \theta_1 \hat{K}_t\big)dt + \big(\hat{K}_t^{\alpha-1} - \hat{K}_{t-}^{\alpha-1}\big)dq_t \\ &= (\alpha-1)\big(\theta_0 \hat{K}_t^{2\alpha-2} - \theta_1 \hat{K}_t^{\alpha-1}\big)dt - \big(1 - (1-\theta_2)^{\alpha-1}\big)\hat{K}_{t-}^{\alpha-1}dq_t, \end{aligned}$$

which implies defining c_1 to c_3 as in (39),

$$\begin{aligned} dr_t &= (\alpha-1)r\left(\frac{1-\tau_i}{1+\tau_k}r_t/\alpha - \theta_1\right)dt + \left((1-\theta_2)^{\alpha-1} - 1\right)r_{t-}dq_t \\ &= c_1 r_t (c_2 - r_t)\, dt + c_3 r_{t-} dq_t. \end{aligned}$$

We use the smooth transformation $\ln r_t$,

$$\ln r_t \overset{\mathcal{D}}{\to} \ln r \quad \text{where} \quad \ln r^* < \ln r_t < \infty, \tag{48}$$

to obtain $d\ln r_t = c_1(c_2 - r_t)dt + \ln(1+c_3)dq_t$, which has the solution

$$\ln r_t - \ln r_{t-\Delta} = \int_{t-\Delta}^{t} c_1(c_2 - r_s)ds + \ln(1+c_3)(q_t - q_{t-\Delta}).$$

Employing the property that $\ln r_t$ and $\ln r_{t-\Delta}$ share the same asymptotic mean as from (48),

$$\begin{aligned}\lim_{t\to\infty} E_0(\ln r_t) - \lim_{t\to\infty} E_0(\ln r_{t-\Delta}) &= c_1c_2\Delta - c_1 \lim_{t\to\infty}\int_{t-\Delta}^{t} E_0(r_s)ds + \ln(1+c_3)\lim_{t\to\infty} E_0(q_\Delta) \\ \Rightarrow \quad E(r) &\equiv \lim_{t\to\infty} E_0(r_t) = \frac{c_1c_2 + \ln(1+c_3)\lambda}{c_1}.\end{aligned}$$

For the second moment, we use the integral equation applying the expectation operator,

$$dE_0(r_t) = c_1\left(c_2E_0(r_t) - E_0(r_t^2)\right)dt + c_3E_0(r_{t-})\lambda dt.$$

Letting $t\to\infty$ we obtain for the integral equation

$$\begin{aligned}\lim_{t\to\infty} E_0(r_t) - \lim_{t\to\infty} E_0(r_{t-\Delta}) &= \lim_{t\to\infty}\int_{t-\Delta}^{t}(c_1c_2 + c_3\lambda)E_0(r_s)ds - \lim_{t\to\infty}\int_{t-\Delta}^{t} c_1E_0(r_s^2)ds \\ \Leftrightarrow \ 0 &= \lim_{t\to\infty} E_0(r_t)(c_1c_2+c_3\lambda)\Delta - \lim_{t\to\infty} E_0(r_t^2)c_1\Delta \\ \Rightarrow \quad E(r^2) &\equiv \lim_{t\to\infty} E_0(r_t^2) = E(r)\frac{c_1c_2+c_3\lambda}{c_1}.\end{aligned}$$

Hence the asymptotic variance, i.e., the variance of the limiting distribution for r_t is

$$Var(r) \equiv \lim_{t\to\infty} Var_0(r_t) = E(r^2) - (E(r))^2 = \frac{c_3\lambda - \ln(1+c_3)\lambda}{c_1}E(r).$$

Note that the variance is proportional to the mean, which seems plausible given the geometric structure of the stochastic differential in (39).

8.7 Moments of growth rates

Because $\ln \hat{K}_t$ is a smooth transformation of a the rental rate of capital, $r_t = \alpha\hat{K}_t^{\alpha-1}L^{1-\alpha}$, the sequence $\{\ln \hat{K}_t\}_{t=t_0}^{\infty}$ converges in distribution to a random variable $\ln \hat{K}$,

$$\ln \hat{K}_t \overset{\mathcal{D}}{\to} \ln \hat{K} \quad \text{where} \quad -\infty < \ln \hat{K}_t < \ln \hat{K}^*.$$

Intuitively, cyclical variables $\ln \hat{K}_t$ and $\ln \hat{K}_{t-\Delta}$ share the same asymptotic mean, which is the mean of the limiting distribution, $E(\ln \hat{K})$. Therefore, defining

$$\begin{aligned}E(\Delta y_t) \equiv \lim_{t\to\infty} E_0(\Delta y_t) &= \lim_{t\to\infty} E_0\big(\ln \hat{K}_t - \ln \hat{K}_{t-\Delta}\big)\alpha + \lim_{t\to\infty} E_0(q_t - q_{t-\Delta})\ln A \\ &= E_0(q_\Delta)\ln A \ = \ \lambda\Delta\ln A\end{aligned}$$

for any $t_0 > 0$, gives the asymptotic mean of output growth rates. Economically, it employs a large sequence of growth rates of length Δ.

Lemma 2 *Given capital rewards as in (39), then*

$$\lim_{t\to\infty} Cov\left(\int_{t-\Delta}^{t}\left(\frac{1-\tau_i}{1+\tau_k}r_s/\alpha\right)ds, q_t - q_{t-\Delta}\right) = 0$$

is asymptotically uncorrelated.

Proof. Observe that from $Cov(aX + bY, Z) = aCov(X, Z) + bCov(Y, Z)$ we have

$$\begin{aligned} Cov\left(\ln \hat{K}_t - \ln \hat{K}_{t-\Delta}, q_t - q_{t-\Delta}\right) &= Cov\left(\int_{t-\Delta}^{t}\left(\frac{1-\tau_i}{1+\tau_k} r_s/\alpha\right) ds, q_t - q_{t-\Delta}\right) \\ &\quad + \ln\left(1-\theta_2\right) Cov\left(q_t - q_{t-\Delta}, q_t - q_{t-\Delta}\right). \end{aligned}$$

Employing Lemma 1 and the property $Var(q_\Delta) = \lambda\Delta$ gives the asymptotic result. ■

Observe that using growth rates of cyclical capital stock in (37) and Lemma 2,

$$\begin{aligned} Var(\hat{y}_t) &= \lim_{t\to\infty} Var_0\left(\frac{1-\tau_i}{1+\tau_k}\int_{t-\Delta}^{t} r_s ds\right) + \alpha^2(\ln(1-\theta_2))^2 \lim_{t\to\infty} Var_0(q_\Delta) \\ &\quad + \alpha^2\ln(1-\theta_2)\lim_{t\to\infty} Cov_0\left(\frac{1-\tau_i}{1+\tau_k}\int_{t-\Delta}^{t} r_s/\alpha ds, q_t - q_{t-\Delta}\right). \end{aligned}$$

Using Lemma 2 we obtain the measure in (38). A similar approach computes the variance of observed output growth rates (cf. app. C.2 for details).

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Web appendix to

On the link between volatility and growth

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The sections of this appendix follow the same structure as in the paper.

A The model

A.1 Deriving the aggregate production function (6)

As written in the text, vintage-specific technologies are given by $Y_j = K_j^\alpha (A^j L_j)^{1-\alpha}$. Labor mobility implies equality of nominal wages for all vintages j, $w_j = w_0\ \forall j$. The wage rate implied by vintage j is given by $w_j = p_t^C (1-\alpha)\left(\frac{K_j}{A^j L_j}\right)^\alpha A^j$. The wage rate of vintage 0 is $w_0 = p_t^C (1-\alpha)\left(\frac{K_0}{A^0 L_0}\right)^\alpha A^0$. Equality of wages for vintages 0 and j implies labor allocation to vintage j relative to vintage 0 of

$$w_0 = w_j \Leftrightarrow \left(\frac{K_0}{A^0 L_0}\right)^\alpha A^0 = \left(\frac{K_j}{A^j L_j}\right)^\alpha A^j \Leftrightarrow \frac{K_0}{A^0 L_0} = A^{\frac{j}{\alpha}} \frac{K_j}{A^j L_j}$$
$$\Leftrightarrow L_j = A^{\frac{j}{\alpha}} \frac{K_j}{A^j} \frac{A^0}{K_0} L_0 = A^{j\frac{1-\alpha}{\alpha}} \frac{K_j}{K_0} L_0. \tag{A.1}$$

Inserting into the labor market clearing condition $\Sigma_{j=0}^q L_j = L$ yields

$$L_0 + A^{\frac{1-\alpha}{\alpha}} \frac{K_1}{K_0} L_0 + ... + A^{q\frac{1-\alpha}{\alpha}} \frac{K_q}{K_0} L_0 = L \Leftrightarrow$$
$$\frac{L_0}{K_0}\left(K_0 + A^{\frac{1-\alpha}{\alpha}} K_1 + ... + A^{q\frac{1-\alpha}{\alpha}} K_q\right) = L \Leftrightarrow L_0 = \frac{K_0}{K_t} L, \tag{A.2}$$

where $K_t \equiv K_0 + A^{\frac{1-\alpha}{\alpha}} K_1 + ... + A^{q\frac{1-\alpha}{\alpha}} K_q \equiv K_0 + BK_1 + ... + B^q K_q$. Inserting (A.2) in (A.1) gives labor allocation to vintage j, $L_j = A^{j\frac{1-\alpha}{\alpha}} LK_j/K_t$. Output of vintage j is therefore $Y_j = K_j^\alpha (A^j L_j)^{1-\alpha} = K_j^\alpha \left(A^{\frac{j}{\alpha}} LK_j/K_t\right)^{1-\alpha} = K_j \left(A^{\frac{j}{\alpha}} L/K_t\right)^{1-\alpha}$. Total output is then given by $Y_t = Y_0 + ... + Y_q = \left(K_0 + K_1 A^{\frac{1-\alpha}{\alpha}} + ... + K_q A^{q\frac{1-\alpha}{\alpha}}\right)(L/K_t)^{1-\alpha} = K_t^\alpha L^{1-\alpha}$.

A.2 The household's budget constraint

Wealth is measured in units of the consumption good, priced at consumer prices. Nominal wealth is given by $(1+\tau_c)p_t^C a_t = \sum_{j=0}^{q+1} k_j v_j$, where k_j is the individual's physical capital stock and v_t is the value of one unit of the capital stock both of vintage j, respectively, and p_t^C is the producer price of the consumption good. Then, real wealth, a_t, is

$$a_t \equiv \frac{1}{1+\tau_c} \sum_{j=0}^{q+1} \frac{k_j v_j}{p_t^C}, \tag{A.3}$$

where q denotes the most advanced vintage. Despite the fact that households cannot own any capital of vintage $q+1$, $k_{q+1} = 0$, the sum applies also to the next vintage to discover. The reason will become clear in a moment.

Households receive net capital payments $(1-\tau_i)\sum_{j=0}^{q+1} p_t^Y w_j^K k_j$, i.e., net dividends per unit of vintage capital j (marginal products) times the amount k_j of all vintages net labor income $(1-\tau_i)p_t^Y w_t$ used for saving and consumption purposes. Nominal savings are

$$s_t = (1-\tau_i)\left(\sum_{j=0}^{q+1} p_t^Y w_j^K k_j + p_t^Y w_t\right) - (1+\tau_c)\, p_t^C c_t.$$

Saving is used for financing research, $(1+\tau_r)p_t^R i_t$, and for accumulating existing capital goods. Households trade capital goods of the most recent vintage q only, the allocation of older capital goods is fixed (households are indifferent about trading in equilibrium). Beside the tax on wealth, τ_a, a fraction $\frac{1-\tau_i}{1+\tau_k}\delta$ of the capital stock depreciates, which implies that only net (and not gross) capital rewards are taxed. Capital held by households follows for *older vintages*

$$dk_j = -\left\{\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right\} k_j dt, \quad j < q. \tag{A.4}$$

The relationship in (A.4) shows that a positive tax on wealth, $\tau_a > 0$, simply increases the rate of effective depreciation. We show below that this tax really applies to wealth a_t and not the number of machines or stocks k_j. A no-arbitrage condition demands that, as long as investment is positive, the value of an installed good equals the price of a new investment good, so that $v_q = (1+\tau_k)p_t^K$. Thus, for the *most recent* vintage, $j = q$, we obtain

$$dk_q = \left\{\frac{s_t - (1+\tau_r)p_t^R i_t}{(1+\tau_k)p_t^K} - \left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right) k_q\right\} dt. \tag{A.5}$$

The household's capital stock k_q of the most advanced vintage q in (A.5) increases when the difference between net income and spending for consumption plus risky investment divided by the effective price v_q of an installed unit of capital, exceeds effective depreciation.

In case of successful research, i.e., $dq_t = 1$, the household obtains the share i_t/R_t of total payoffs κ_t. Therefore, for the *next vintage*, $j = q+1$, we obtain

$$dk_{q+1} = i_t/R_t \kappa_t dq_t. \tag{A.6}$$

In words, the individual payoff depends on individual investment i_t relative to total investment R_t into the successful project. After that, (A.5) applies to vintage $q+1$.

As different vintages are perfect substitutes in production, from (7) prices are linked by

$$(1+\tau_k)p_t^K = v_q = B^{q-j}v_j, \quad j \leq q. \tag{A.7}$$

Immediately after the innovation, the price of older vintages relative to the producer price of the consumption good using $p_t^K = p_t^C$ from (5) falls by $v_j/p_t^C = (1+\tau_k)B^{-(q-j)}$, $j \leq q$.

Assuming that vintage prices generally evolve as

$$d(v_j/p_t^C) = a_j v_j/p_t^C dt + c_j v_j/p_{t-}^C dq_t, \tag{A.8}$$

the deterministic part must be zero, i.e., $a_j = 0$ for all vintages $j \leq q$. When research is successful, the price of a unit of given vintage j in terms of the consumption good drops by $v_j/p_t^C = (1+\tau_k)B^{-(q+1-j)}$ from (A.7). Thus, using (A.8) and $s \equiv (B-1)/B$ we obtain

$$c_j = \frac{d(v_j/p_t^C)}{v_j/p_{t-}^C} = \frac{v_j/p_t^C - v_j/p_{t-}^C}{v_j/p_{t-}^C} = \frac{(1+\tau_k)B^{-(q+1-j)} - (1+\tau_k)B^{-(q-j)}}{(1+\tau_k)B^{-(q-j)}} = -s < 0,$$

which is identical for all vintages $j \leq q$, and prices therefore evolve according to

$$d(v_j/p_t^C) = -s v_j/p_{t-}^C dq_t, \quad j \leq q. \tag{A.9}$$

This equation reflects the devaluation (henceforth economic depreciation) of older vintages relative to the consumption good when a new vintage has been developed.

The budget constraint can now be derived by computing the differential

$$da = \frac{1}{1+\tau_c}\sum_{j=0}^{q+1} d(k_j v_j/p_t^C). \tag{A.10}$$

Note that is implicitly assumed that tax rates are considered to be constant. For *older* vintages $0 < j < q$, we obtain using (A.4), (A.9) and applying Itô's formula

$$\begin{aligned} d(k_j v_j/p_t^C) &= -\frac{v_j}{p_t^C}\left\{\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right\} k_j dt + \left\{\left(\frac{v_j}{p_{t-}^C} - s\frac{v_j}{p_{t-}^C}\right)k_j - \frac{v_j}{p_{t-}^C}k_j\right\} dq_t \\ &= -\left\{\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right\} k_j v_j/p_t^C dt - s k_j v_j/p_{t-}^C dq_t. \end{aligned} \tag{A.11}$$

Similarly, for the *most advanced* vintage, q, we use (A.5), (A.9) and apply Itô's formula,

$$\begin{aligned} d(k_q v_q/p_t^C) &= \frac{v_q}{p_t^C}\left\{\frac{s_t - (1+\tau_r)p_t^R i_t}{(1+\tau_k)p_t^K} - \left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)k_q\right\} dt - s\frac{v_q}{p_{t-}^C}k_q dq_t \\ &= \left\{\frac{s_t}{p_t^C} - (1+\tau_r)i_t - \left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)k_q v_q/p_t^C\right\} dt - s k_q v_q/p_{t-}^C dq_t, \end{aligned} \tag{A.12}$$

where we used (5) in the last step. Finally for the *next* vintage $q+1$ to come, using equation (A.6) with $v_{q+1}/p_t^C = (1+\tau_k)$ for the prototype after successful research, yields

$$d(k_{q+1}v_{q+1}/p_t^C) = v_{q+1}/p_t^C i_t/R_t\kappa_t dq_t = (1+\tau_k)i_t/R_t\kappa_t dq_t. \tag{A.13}$$

This is in accordance to the definition of *real* wealth in (A.3) since it denotes the consumption goods that can be exchanged for the prototype. Hence, as we want to include the evolution of κ_t, assets a_t need to equal the sum over all vintages including the not yet existing one.

Summarizing (A.11) to (A.13), the budget constraint (A.10) becomes

$$da_t = \frac{1}{1+\tau_c}\sum_{j=0}^{q-1}\left(-\left\{\frac{1-\tau_i}{1+\tau_k}\delta+\tau_a\right\}k_j v_j/p_t^C dt - sk_j v_j/p_{t-}^C dq_t\right) + \frac{1+\tau_k}{1+\tau_c}i_t/R_t\kappa_t dq_t$$
$$+\frac{1}{1+\tau_c}\left\{s_t/p_t^C-(1+\tau_r)i_t-\left(\frac{1-\tau_i}{1+\tau_k}\delta+\tau_a\right)k_q v_q/p_t^C\right\}dt-\frac{1}{1+\tau_c}sk_q v_q/p_{t-}^C dq_t.$$

Using the definition of *real* wealth (A.3) and inserting s_t yields

$$da_t = \left\{\frac{1-\tau_i}{1+\tau_c}\left(\sum_{j=0}^{q+1}w_j^K k_j + w_t\right) - c_t - \frac{1+\tau_r}{1+\tau_c}i_t\right\}dt - a_t\left\{\frac{1-\tau_i}{1+\tau_k}\delta+\tau_a\right\}dt$$
$$+\left\{\frac{1+\tau_k}{1+\tau_c}i_t/R_t\kappa_t - sa_{t-}\right\}dq_t. \tag{A.14}$$

With value marginal product, $w_j^K = Y_K B^j$ from equation (7) and (6), as well as inserting $B^j = B^q v_j/v_q$ from (A.7) with $v_q = (1+\tau_k)p_K$ we obtain

$$da_t = \left\{\frac{1-\tau_i}{1+\tau_c}\left(\sum_{j=0}^{q+1}B^q Y_K\frac{v_j k_j}{(1+\tau_k)p_t^K} + w_t\right) - c_t - \frac{1+\tau_r}{1+\tau_c}i_t\right\}dt - a_t\left\{\frac{1-\tau_i}{1+\tau_k}\delta+\tau_a\right\}dt$$
$$+\left\{\frac{1+\tau_k}{1+\tau_c}i_t/R_t\kappa_t - sa_{t-}\right\}dq_t.$$

Finally, using the definition of *real* wealth (A.3) and the definition of r_t in (14) gives

$$da_t = \left\{\left(\frac{1-\tau_i}{1+\tau_k}(r_t-\delta)-\tau_a\right)a_t + \frac{1-\tau_i}{1+\tau_c}w_t - c_t - \frac{1+\tau_r}{1+\tau_c}i_t\right\}dt + \left\{\frac{1+\tau_k}{1+\tau_c}i_t/R_t\kappa_t - sa_{t-}\right\}dq_t.$$

A.3 Total wealth and government expenditure

We show that aggregating the budget constraints of households yields the aggregate resource constraint. It also gives us the government's budget constraint.

Starting by summing up the budget constraint (13) of the representative consumer using $\sum_{i=1}^{L} a_t = La_t$, we obtain

$$dLa_t = \left\{\left(\frac{1-\tau_i}{1+\tau_k}(r_t-\delta)-\tau_a\right)La_t + \frac{1-\tau_i}{1+\tau_c}w_t L - \frac{1+\tau_r}{1+\tau_c}R_t - C_t\right\}dt$$
$$+\left\{\frac{1+\tau_k}{1+\tau_c}\kappa_t - sLa_{t-}\right\}dq_t,$$

where C_t and R_t denote $C_t = Lc_t$ and $R_t = Li_t$, respectively. Using the definition of wealth in (A.3) together with (A.7), we obtain total wealth as

$$La_t = \frac{1}{1+\tau_c}\sum_{j=0}^{q+1}\frac{k_j v_j}{p_t^C}L = \frac{1+\tau_k}{1+\tau_c}B^{-q}\sum_{j=0}^{q+1}K_j B^j = \frac{1+\tau_k}{1+\tau_c}B^{-q}K_t \equiv K_t^{obs},$$

where we used the definition of the capital index from (7) in the last step. Inserting yields

$$d\left(\frac{1+\tau_k}{1+\tau_c}B^{-q}K_t\right) = \left\{\left(\frac{1-\tau_i}{1+\tau_k}(r_t-\delta)-\tau_a\right)\frac{1+\tau_k}{1+\tau_c}B^{-q}K_t + \frac{1-\tau_i}{1+\tau_c}w_tL - \frac{1+\tau_r}{1+\tau_c}R_t - C_t\right\}dt + \left\{\frac{1+\tau_k}{1+\tau_c}\kappa_t - s\frac{1+\tau_k}{1+\tau_c}B^{-q}K_{t-}\right\}dq_t$$

$$\Leftrightarrow\ d\left(B^{-q}K_t\right) = \left\{\left(\frac{1-\tau_i}{1+\tau_k}(r_t-\delta)-\tau_a\right)B^{-q}K_t + \frac{1-\tau_i}{1+\tau_k}w_tL - \frac{1+\tau_r}{1+\tau_k}R_t - \frac{1+\tau_c}{1+\tau_k}C_t\right\}dt + \left\{\kappa_t - sB^{-q}K_{t-}\right\}dq_t.$$

Using Itô's formula we obtain

$$\begin{aligned} dK_t &= \left\{\left(\frac{1-\tau_i}{1+\tau_k}(r_t-\delta)-\tau_a\right)B^{-q}K_t + \frac{1-\tau_i}{1+\tau_k}w_tL - \frac{1+\tau_r}{1+\tau_k}R_t - \frac{1+\tau_c}{1+\tau_k}C_t\right\}B^q dt \\ &\quad + \left\{\left(B^{-q}K_{t-}+\kappa_t - sB^{-q}K_{t-}\right)B^{q+1} - \left(B^{-q}K_{t-}\right)B^q\right\}dq_t \\ &= \left\{\left(\frac{1-\tau_i}{1+\tau_k}(r_t-\delta)-\tau_a\right)B^{-q}K_t + \frac{1-\tau_i}{1+\tau_k}w_tL - \frac{1+\tau_r}{1+\tau_k}R_t - \frac{1+\tau_c}{1+\tau_k}C_t\right\}B^q dt \\ &\quad + \left\{(1-s)BK_{t-} + B^{q+1}\kappa_t - K_{t-}\right\}dq_t. \end{aligned}$$

Inserting factor rewards from (14) and $s=(B-1)/B$ yields

$$\begin{aligned} dK_t &= B^q\left\{\left(\frac{1-\tau_i}{1+\tau_k}\left(B^qY_K-\delta\right)-\tau_a\right)B^{-q}K_t + \frac{1-\tau_i}{1+\tau_k}Y_LL\right. \\ &\quad \left. -\frac{1+\tau_r}{1+\tau_k}R_t - \frac{1+\tau_c}{1+\tau_k}C_t\right\}dt + B^{q+1}\kappa_t dq_t. \end{aligned} \tag{A.15}$$

Now observe that

$$\frac{1-\tau_i}{1+\tau_k}Y_KK_t + \frac{1-\tau_i}{1+\tau_k}Y_LL = Y_KK_t + Y_LL - \frac{\tau_k+\tau_i}{1+\tau_k}\left(Y_KK_t+Y_LL\right) = Y_t - \frac{\tau_k+\tau_i}{1+\tau_k}Y_t,$$

where we used Euler's theorem, $Y_t = Y_KK_t + Y_LL$. Moreover, observe that

$$\begin{aligned} &-\frac{1-\tau_i}{1+\tau_k}\delta B^{-q}K_t - \frac{1+\tau_r}{1+\tau_k}R_t - \frac{1+\tau_c}{1+\tau_k}C_t \\ =\ & -\delta B^{-q}K_t - R_t - C_t + \frac{\tau_i+\tau_k}{1+\tau_k}\delta B^{-q}K_t - \frac{\tau_r-\tau_k}{1+\tau_k}R_t - \frac{\tau_c-\tau_k}{1+\tau_k}C_t. \end{aligned}$$

Therefore, the term in brackets from (A.15) can be simplified to,

$$\begin{aligned} &\left(\frac{1-\tau_i}{1+\tau_k}\left(B^qY_K-\delta\right)-\tau_a\right)B^{-q}K_t + \frac{1-\tau_i}{1+\tau_k}Y_LL - \frac{1+\tau_r}{1+\tau_k}R_t - \frac{1+\tau_c}{1+\tau_k}C_t \\ =\ & Y_t - \frac{\tau_i+\tau_k}{1+\tau_k}Y_t - (\tau_a+\delta)B^{-q}K_t - R_t - C_t + \frac{\tau_i+\tau_k}{1+\tau_k}\delta B^{-q}K_t - \frac{\tau_r-\tau_k}{1+\tau_k}R_t - \frac{\tau_c-\tau_k}{1+\tau_k}C_t \\ \equiv\ & Y_t - \delta B^{-q}K_t - R_t - C_t - G_t. \end{aligned} \tag{A.16}$$

The last equality defines total tax revenues and thus government expenditures,

$$G_t \equiv \frac{\tau_i+\tau_k}{1+\tau_k}Y_t + \left(\tau_a - \frac{\tau_i+\tau_k}{1+\tau_k}\delta\right)B^{-q}K_t + \frac{\tau_r-\tau_k}{1+\tau_k}R_t + \frac{\tau_c-\tau_k}{1+\tau_k}C_t. \tag{A.17}$$

Inserting (A.16) into (A.15) gives

$$\begin{aligned} dK_t &= \left\{Y_t - \delta B^{-q} K_t - R_t - C_t - G_t\right\} B^q dt + B^{q+1}\kappa_t dq_t \\ &= \left\{B^q I_t - \delta K_t\right\} dt + B^{q+1}\kappa_t dq_t, \end{aligned}$$

which is (8). Aggregation works for G_t as defined in (A.17). Equation (A.17) is convincing, if it can be rewritten in a meaningful way. Starting at

$$\begin{aligned} (1+\tau_k)\, G_t &\equiv \tau_k Y_t + \tau_i Y_t + (1+\tau_k)\,\tau_a B^{-q} K_t - (\tau_i + \tau_k)\,\delta B^{-q} K_t \\ &\quad + (\tau_r - \tau_k)\, R_t + (\tau_c - \tau_k)\, C_t. \end{aligned}$$

As $\tau_k G_t = \tau_k \left(Y_t - I_t - R_t - C_t\right)$ from (4), we obtain

$$\begin{aligned} G_t &= \tau_k I_t + \tau_i Y_t + (1+\tau_k)\,\tau_a B^{-q} K_t - (\tau_i + \tau_k)\,\delta B^{-q} K_t + \tau_r R_t + \tau_c C_t \\ &= \tau_i (Y_t - \delta B^{-q} K_t) + \tau_k (I_t - \delta B^{-q} K_t) + \tau_r R_t + \tau_c C_t + \tau_a \,(1+\tau_k)\, B^{-q} K_t, \end{aligned}$$

i.e., government revenue in units of the consumption good before taxation.

B Equilibrium dynamics

B.1 The maximized Bellman equation

The value of an optimal program of (10) is defined by

$$V(a_0, q_0) = \max_{\{c_t, i_t\}_{t=0}^{\infty}} \{U_0\},$$

which denotes the present discounted value of utility evaluated along the optimal program. Itô's formula (or change of variables, cf. Sennewald, 2007) yields

$$\begin{aligned} dV_t &= V_a \left\{ \left(\frac{1-\tau_i}{1+\tau_k}(r_t - \delta) - \tau_a \right) a_t + \frac{1-\tau_i}{1+\tau_c} w_t - c_t - \frac{1+\tau_r}{1+\tau_c} i_t \right\} dt \\ &\quad + \left\{ V(a_t, q_t) - V(a_{t-}, q_{t-}) \right\} dq_t. \end{aligned}$$

With $E_0(dq_t) = \lambda_t dt$, the Bellman equation reads

$$\begin{aligned} \rho V(a_0, q_0) = \max_{\{c_0, i_0\}} \Bigg\{ & u(c_0) + V_a \left\{ \left(\frac{1-\tau_i}{1+\tau_k}(r_0 - \delta) - \tau_a \right) a_0 + \frac{1-\tau_i}{1+\tau_c} w_0 - c_0 - \frac{1+\tau_r}{1+\tau_c} i_0 \right\} \\ & + \lambda_t \left\{ V(a_0, q_0) - V(a_{0-}, q_{0-}) \right\} \Bigg\}, \end{aligned} \tag{B.18}$$

where the level of q_t and a_t immediately after a jump is $q_t = q_{t-} + 1$ and

$$a_t = (1-s) a_{t-} + \frac{1+\tau_k}{1+\tau_c} \frac{i_t}{R_t} \kappa_t. \tag{B.19}$$

The first-order condition for consumption reads

$$u'(c_0) = V_a(a_0, q_0), \tag{B.20}$$

whereas the first-order condition for the risky investment is

$$\frac{1+\tau_r}{1+\tau_c} V_{a_{0-}}(a_{0-}, q_{0-}) = \lambda_t V_{a_0}(a_0, q_0) \frac{1+\tau_k}{1+\tau_c} \frac{\kappa_0}{R_0}. \tag{B.21}$$

Inserting (B.20) twice and rearranging gives

$$u'(c_{0-}) = \lambda_t \frac{1+\tau_k}{1+\tau_r} \frac{\kappa_0}{R_0} u'(c_0). \tag{B.22}$$

Economically, research expenditures, i_t, are chosen such that the ratio of marginal utilities from consumption an instant before and after a jump, $u'(c_{t-})/u'(c_t)$ equals to the ratio of expected payoffs of the risky research, $\lambda_t(1+\tau_k)\kappa_t i_t/R_t$, divided by its cost, $(1+\tau_r)i_t$.

The first-order conditions (B.20) and (B.21) make optimal controls a function of the state variables, $c_t = c(a_t, q_t)$ and $i_t = i(a_t, q_t)$. Hence, the maximized Bellman equation is

$$\begin{aligned} \rho V(a_t, q_t) = \; & V_a \left\{ \left(\frac{1-\tau_i}{1+\tau_k}(r_t - \delta) - \tau_a \right) a_t + \frac{1-\tau_i}{1+\tau_c} w_t - c(a_t) - \frac{1+\tau_r}{1+\tau_c} i(a_t) \right\} \\ & + u(c(a_t)) + \lambda_t \left\{ V(a_t, q_t) - V(a_{t-}, q_{t-}) \right\}. \end{aligned} \tag{B.23}$$

B.2 The rule of optimal consumption

We start with the maximized Bellman equation in (B.23). Using the envelope theorem, the derivative with respect to the state variable reads[16]

$$\begin{aligned} \rho V_a = \; & V_{aa} \left\{ \left(\frac{1-\tau_i}{1+\tau_k}(r_t - \delta) - \tau_a \right) a_t + \frac{1-\tau_i}{1+\tau_c} w_t - c(a_t) - \frac{1+\tau_r}{1+\tau_c} i(a_t) \right\} \\ & + \left(\frac{1-\tau_i}{1+\tau_k}(r_t - \delta) - \tau_a \right) V_a + \lambda_t \left\{ V_{a_{t-}}(a_t, q_t) - V_{a_{t-}}(a_{t-}, q_{t-}) \right\}. \end{aligned} \tag{B.24}$$

Now observe that from (B.19), $V_{a_{t-}}(a_t, q_t) = V_{a_t}(1-s)$. Rearranging gives

$$\begin{aligned} & \left(\rho - \frac{1-\tau_i}{1+\tau_k}(r_t - \delta) + \tau_a + \lambda_t \right) V_a - (1-s)\lambda_t V_{a_t} = \\ & V_{aa} \left\{ \left(\frac{1-\tau_i}{1+\tau_k}(r_t - \delta) - \tau_a \right) a_t + \frac{1-\tau_i}{1+\tau_c} w_t - c_t - \frac{1+\tau_r}{1+\tau_c} i_t \right\}. \end{aligned} \tag{B.25}$$

Computing the differential of the costate variable, using the budget constraint (13) and Itô's formula (change of variables) gives

$$\begin{aligned} dV_a = \; & V_{aa} \left\{ \left(\frac{1-\tau_i}{1+\tau_k}(r_t - \delta) - \tau_a \right) a_t + \frac{1-\tau_i}{1+\tau_c} w_t - c_t - \frac{1+\tau_r}{1+\tau_c} i_t \right\} dt \\ & + \left\{ V_{a_t}(a_t, q_t) - V_{a_{t-}}(a_{t-}, q_{t-}) \right\} dq_t. \end{aligned}$$

[16] The notion of V_a and $V_{a_{t-}}$ are equivalent, the term $V_{a_{t-}}$ is used if necessary for clarity.

Inserting equation (B.25) yields the evolution of the costate variable as

$$dV_a = \left(\rho - \frac{1-\tau_i}{1+\tau_k}(r_t - \delta) + \tau_a + \lambda_t\right) V_a - (1-s)\,\lambda_t V_{a_t} dt + \left\{V_{a_t}(a_t, q_t) - V_{a_{t-}}(a_{t-}, q_{t-})\right\} dq_t.$$

Dividing by the costate an instant before the jump, V_a, we obtain

$$\frac{dV_a}{V_a} = \left\{\rho - \frac{1-\tau_i}{1+\tau_k}(r_t - \delta) + \tau_a + \lambda_t - (1-s)\,\lambda_t \frac{V_{a_t}(a_t, q_t)}{V_{a_{t-}}(a_{t-}, q_{t-})}\right\} dt + \left\{\frac{V_{a_t}(a_t, q_t)}{V_{a_{t-}}(a_{t-}, q_{t-})} - 1\right\} dq_t.$$

The first-order condition for consumption (B.20) yields $dV_a = du'(c_t)$. Inserting this term as well as using the first order condition (B.20) gives the optimal rule as

$$\frac{du'(c_{t-})}{u'(c_{t-})} = \left(\rho - \frac{1-\tau_i}{1+\tau_k}(r_t - \delta) + \tau_a + \lambda_t - (1-s)\,\lambda_t \frac{u'(c_t)}{u'(c_{t-})}\right) dt + \left(\frac{u'(c_t)}{u'(c_{t-})} - 1\right) dq_t. \tag{B.26}$$

Note that the ratio of marginal utilities before and after a jump is determined by the first-order condition of risky investment (B.22),

$$\frac{u'(c_t)}{u'(c_{t-})} = \frac{R_t}{\kappa_t}\frac{1+\tau_r}{1+\tau_k}\lambda_t^{-1} = \frac{1+\tau_r}{1+\tau_k}\lambda_t^{\frac{\gamma}{1-\gamma}} D/\kappa_0 = \xi^{-\sigma}, \tag{B.27}$$

where we inserted $R_t = \lambda_t^{\frac{1}{1-\gamma}} D_t$ from (1), and used the assumptions of (15) and (16).

Using (11) and Itô's formula, we may write the rule for optimal consumption (B.26) as

$$\begin{aligned}
dC_t &= -\frac{1}{\sigma}\left(\rho - \frac{1-\tau_i}{1+\tau_k}(r_t - \delta) + \tau_a + \lambda_t - (1-s)\,\lambda_t \xi^{-\sigma}\right) C_t^{-\sigma}\left(C_t^{-\sigma}\right)^{-\frac{1}{\sigma}-1} dt \\
&\quad + \left(\left(C_{t-}^{-\sigma} + \left\{\frac{u'(c_t)}{u'(c_{t-})} - 1\right\} C_{t-}^{-\sigma}\right)^{-\frac{1}{\sigma}} - C_{t-}\right) dq_t \\
&= -\frac{1}{\sigma}\left(\rho - \frac{1-\tau_i}{1+\tau_k}(r_t - \delta) + \tau_a + \lambda_t - (1-s)\,\lambda_t \xi^{-\sigma}\right) C_t dt + \left(C_t - C_{t-}\right) dq_t.
\end{aligned}$$

In terms of cyclical consumption, we obtain the Euler equation

$$d\hat{C}_t = -\frac{1}{\sigma}\left(\rho - \frac{1-\tau_i}{1+\tau_k}(r_t - \delta) + \tau_a + \lambda_t - (1-s)\,\lambda_t \xi^{-\sigma}\right) \hat{C}_t dt + \left(\hat{C}_t - \hat{C}_{t-}\right) dq_t. \tag{B.28}$$

B.3 Proof of Theorem 1

The idea of this proof is to first assume that $\hat{C}_t = \Psi \hat{K}_t$ and then derive conditions under which this holds. We first derive the evolution of the cyclical capital index, and split up the proof by sequentially considering the jump and the deterministic component of the SDE.

Given the capital accumulation constraint in (8) with the market clearing condition (4) and using Itô's formula (change of variables) for $\hat{K}_t$ defined in (17), we obtain

$$\begin{aligned}
d\hat{K}_t &= \left((Y_t - C_t - R_t - G_t)B^q - \delta K_t\right) A^{-\frac{q}{\alpha}} dt + \left(\left(K_{t-} + B^{q+1}\kappa_t\right) A^{-\frac{q+1}{\alpha}} - A^{-\frac{q}{\alpha}} K_{t-}\right) dq_t \\
&= \left(\hat{Y}_t - \hat{C}_t - \hat{R}_t - \hat{G}_t - \delta \hat{K}_t\right) dt + \left(A^{-\frac{1}{\alpha}} + A^{-1}\kappa - 1\right) \hat{K}_{t-} dq_t,
\end{aligned}$$

where we inserted (16) and used the definition of cyclical variables from (17), $\hat{X}_t = A^{-q}X_t$ for $X_t \in (Y_t, C_t, R_t, G_t)$. With the definition of the parameter ξ in Theorem 1, we obtain

$$d\hat{K}_t = \left(\hat{Y}_t - \hat{C}_t - \hat{R}_t - \hat{G}_t - \delta\hat{K}_t\right) dt + \left(A^{-1}\xi - 1\right) \hat{K}_{t-}dq_t. \tag{B.29}$$

Considering the jump term of the SDE in (B.29), an instant after successful research, the level of cyclical capital is $\hat{K}_t = A^{-1}\xi\hat{K}_{t-}$. This implies that in the linear case of $\hat{C}_t = \Psi\hat{K}_t$, consumption jumps by $\hat{C}_t/\hat{C}_{t-} = A^{-1}\xi$ as well. From (B.27), we obtain another expression for the jump of cyclical consumption using $u'(c_t) = c_t^{-\sigma}$ from (11) and $C_t = Lc_t$,

$$C_t/C_{t-} = A\hat{C}_t/\hat{C}_{t-} = \left(\frac{1+\tau_r}{1+\tau_k}\lambda_t^{\frac{\gamma}{1-\gamma}}D/\kappa\right)^{-\frac{1}{\sigma}}.$$

Thus, we obtain a constant arrival rate by solving for $\lambda = \lambda_t$ as

$$\xi = \left(\frac{1+\tau_r}{1+\tau_k}\lambda^{\frac{\gamma}{1-\gamma}}D/\kappa\right)^{-\frac{1}{\sigma}} \quad\Leftrightarrow\quad \lambda = \left(\frac{1+\tau_k}{1+\tau_r}\frac{\kappa}{D}\xi^{-\sigma}\right)^{\frac{1-\gamma}{\gamma}},$$

which is (22). Using (1) and $R_t = A^q\hat{R}_t$ gives

$$\hat{R}_t = \lambda^{\frac{1}{1-\gamma}}\hat{K}_tD = \frac{1+\tau_k}{1+\tau_r}\kappa\lambda\xi^{-\sigma}\hat{K}_t \;\equiv\; \Gamma\hat{K}_t. \tag{B.30}$$

Optimal consumption $\hat{C}_t = \Psi\hat{K}_t$ also requires that consumption and capital grow at the same rates over the cycle. In other words, the SDEs or growth rates $d\hat{C}_t/\hat{C}_t$ and $d\hat{K}_t/\hat{K}_t$ resulting from (B.28) and (B.29), respectively, must share the same deterministic part,

$$-\frac{1}{\sigma}\left(\rho - \frac{1-\tau_i}{1+\tau_k}(r_t-\delta) + \tau_a + \lambda - (1-s)\lambda\xi^{-\sigma}\right) = \frac{\hat{Y}_t - \hat{C}_t - \hat{R}_t - \hat{G}_t - \delta\hat{K}_t}{\hat{K}_t}.$$

Inserting capital rewards from (14), $r_t = B^qY_K = \alpha\hat{Y}_t/\hat{K}_t$ and multiply by $\sigma\hat{K}_t$ to obtain

$$\frac{1-\tau_i}{1+\tau_k}\alpha\hat{Y}_t = \left(\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a + \omega\right)\hat{K}_t + \sigma\left(\hat{Y}_t - \hat{C}_t - \hat{R}_t - \hat{G}_t - \delta\hat{K}_t\right),$$

where we defined $\omega \equiv \rho + \lambda - (1-s)\lambda\xi^{-\sigma}$. Inserting $\hat{C}_t = \Psi\hat{K}_t$ associated with $\hat{R}_t = \Gamma\hat{K}_t$ from (B.30), and using $\hat{G}_t = A^{-q}G_t$ from (A.17) the condition can be simplified to

$$\frac{1-\tau_i}{1+\tau_k}(\alpha-\sigma)\hat{Y}_t = \left\{\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a + \omega - \sigma\left(\tau_a + \frac{1+\tau_r}{1+\tau_k}\Gamma + \frac{1+\tau_c}{1+\tau_k}\Psi + \frac{1-\tau_i}{1+\tau_k}\delta\right)\right\}\hat{K}_t. \tag{B.31}$$

Hence, (B.31) holds if the expression in front of $\hat{Y}$ holds, requiring $\alpha = \sigma$ and if the expression in front of $\hat{K}$ in (B.31) holds, requiring

$$\frac{1-\tau_i}{1+\tau_k}\delta + \tau_a + \omega = \sigma\left(\tau_a + \frac{1+\tau_r}{1+\tau_k}\Gamma + \frac{1+\tau_c}{1+\tau_k}\Psi + \frac{1-\tau_i}{1+\tau_k}\delta\right).$$

Solving for Ψ and inserting $\omega = \rho + \lambda - (1-s)\lambda\xi^{-\sigma}$ yields the expression for Ψ in (19).

C Volatility measures

C.1 The second moment in the long run

We show that the second moment of cyclical output, $\hat{m}_t^2$, is constant in the long run. From (44), the ODE for the second moment is $d\hat{m}_t^2 = \left(\theta_0 \hat{m}_t^1 - (\theta_1 + \lambda\theta_2)\, \hat{m}_t^2\right) dt$. Inserting the solution of the first moment from (42) gives for our parametric restriction $\alpha = 0.5$,

$$d\hat{m}_t^2 = \left(\theta_0 \left(e^{-(\alpha\theta_1 + \lambda\vartheta_2)t} \left(\hat{m}_0^1 - \hat{m}^1\right) + \hat{m}^1\right) - (\theta_1 + \lambda\theta_2)\, \hat{m}_t^2\right) dt.$$

Solving this deterministic differential equation with time-varying coefficients gives

$$\hat{m}_t^2 = e^{-(\theta_1+\lambda\theta_2)t}\hat{m}_0^2 + \int_0^t \theta_0 \left(e^{-(\alpha\theta_1+\lambda\vartheta_2)s} \left(\hat{m}_0^1 - \hat{m}^1\right) + \hat{m}^1\right) e^{(\theta_1+\lambda\theta_2)(s-t)} ds.$$

Computing the integral yields

$$\hat{m}_t^2 \;=\; e^{-(\theta_1+\lambda\theta_2)t} m_0^2 + \theta_0 \left(\hat{m}_0^1 - \hat{m}^1\right) \frac{e^{-(\alpha\theta_1+\lambda\vartheta_2)t} - e^{-(\theta_1+\lambda\theta_2)t}}{\alpha\theta_1 + \lambda(1-\theta_2)^\alpha - \lambda(1-\theta_2)} + \theta_0 \hat{m}^1 \frac{1 - e^{-(\theta_1+\lambda\theta_2)t}}{\theta_1 + \lambda\theta_2}.$$

Since $(1-\theta_2)^\alpha = \left(A^{-1}\xi\right)^\alpha < 1$, the exponential terms decrease in t. With t sufficiently large, those terms vanish and the expression converges to (34).

C.2 Derivation of (27)

According to the detrending rule (17), we may write logarithmic output as

$$\ln Y_t \;=\; \alpha \ln K_t + (1-\alpha)\ln L = \alpha \ln \hat{K}_t + (1-\alpha)\ln L + q_t \ln A, \tag{C.32}$$

i.e., we split our time series $\ln Y_t$ into a trend component, $q_t \ln A$, and a stationary component, $\alpha \ln \hat{K}_t + (1-\alpha)L$.[17] Both the trend component and the stationary component are stochastic. Even though our model is formulated in continuous time, we can relate our trend component to a discrete-time random walk as $q_t \equiv q_{t-\Delta} + \Delta q_t$, where $\Delta q_t \sim (\lambda\Delta, \lambda\Delta)$. The trend component $q_t \ln A$ has a unit root and cyclical capital $\hat{K}_t$ is stationary by construction.

Let the growth rates per unit of time be $\Delta y_t \equiv \ln Y_t - \ln Y_{t-\Delta}$, from (C.32) we obtain

$$\Delta y_t = \; \alpha\left(\ln \hat{K}_t - \ln \hat{K}_{t-\Delta}\right) + (q_t - q_{t-\Delta})\ln A \;=\; \Delta\hat{y}_t + \Delta q_t \ln A. \tag{C.33}$$

Using Lemma 1 and (26), we define our second measure based on output growth rates as

$$\begin{aligned} Var(\Delta y_t) \;&\equiv\; Var(\Delta\hat{y}_t) + \lim_{t\to\infty} Cov_0(\ln \hat{K}_t - \ln \hat{K}_{t-\Delta}, q_t - q_{t-\Delta}) + Var(\Delta q_t \ln A) \\ &=\; Var(\Delta\hat{y}_t) + 2\alpha \ln A \ln(1-\theta_2)\lambda\Delta + (\ln A)^2\lambda\Delta \qquad\qquad (C.34) \\ &=\; \tfrac{\alpha^2}{1-\alpha}\left(\tfrac{1-(1-\theta_2)^{1-\alpha}}{(1-\theta_2)^{1-\alpha}} + (1-\alpha)\ln(1-\theta_2)\right)\left(\theta_1 - \ln(1-\theta_2)\lambda\right)\lambda\Delta^2 \\ &\quad + \left(\alpha\ln(1-\theta_2) + \ln A\right)^2 \lambda\Delta. \end{aligned}$$

This is expression (27) used in the main text.

[17] Other models of endogenous fluctuations and growth are of a deterministic nature. An exception is Bental and Peled (1996) who first studied endogenous fluctuations and growth. Unfortunately, their model is fairly complex making an explicit analysis of stochastic properties of trends and cycles a difficult task.

C.3 Proof of Lemma 1

Observe that the covariance is

$$
\begin{aligned}
&\lim_{t\to\infty} Cov_0\left(\ln \hat{K}_t - \ln \hat{K}_{t-\Delta}, q_t - q_{t-\Delta}\right) \\
&\quad = \lim_{t\to\infty} E_0\left(\int_{t-\Delta}^{t} d(\ln \hat{K}_s)\left(\int_{t-\Delta}^{t} dq_s - \lambda\Delta\right)\right) \\
&\quad = \lim_{t\to\infty} E_0\left(\int_{t-\Delta}^{t} \left((\theta_0 \hat{K}_s^{\alpha-1} - \theta_1)ds + \ln(1-\theta_2)\, dq_s\right)\int_{t-\Delta}^{t} dq_s\right) \\
&\quad = \lim_{t\to\infty} E_0\left(\int_{t-\Delta}^{t} \frac{1-\tau_i}{1+\tau_k} r_t/\alpha ds \int_{t-\Delta}^{t} dq_s - \theta_1 \Delta q_\Delta + \ln(1-\theta_2)\,(q_\Delta)^2\right) \\
&\quad = \lim_{t\to\infty} E_0\left(\int_{t-\Delta}^{t} \frac{1-\tau_i}{1+\tau_k} r_t/\alpha ds \int_{t-\Delta}^{t} dq_s\right) - \lim_{t\to\infty} E_0\left(\theta_1 \Delta q_\Delta - \ln(1-\theta_2)\,(q_\Delta)^2\right) \\
&\quad = \frac{(1-\alpha)\theta_1 - (1-\alpha)\ln(1-\theta_2)\lambda}{1-\alpha}\Delta\lambda\Delta - \theta_1\Delta\lambda\Delta + \ln(1-\theta_2)\,\lambda\Delta(1+\lambda\Delta) \\
&\quad = \theta_1\lambda\Delta^2 - \ln(1-\theta_2)(\lambda\Delta)^2 - \theta_1\Delta\lambda\Delta + \ln(1-\theta_2)\,\lambda\Delta(1+\lambda\Delta) \\
&\quad = \ln(1-\theta_2)\,\lambda\Delta,
\end{aligned}
$$

where we used the property that $E(q_\Delta^2) = \lambda\Delta(1+\lambda\Delta)$. Moreover, we used that

$$\int_{t-\Delta}^{t}\int_{t-\Delta}^{t}\left(\frac{1-\tau_i}{1+\tau_k} r_s/\alpha\right) ds d(q_u - \lambda u)$$

is a martingale (cf. García and Griego, 1994, Theorem 5.3).

D Volatility and taxation

D.1 Theoretical findings

For reading convenience, we compute the derivative of the parameter governing the speed of convergence, θ_1, with respect to taxes for later use. Insert (22) into the definition of θ_1,

$$\theta_1 = \frac{1}{\sigma}\left(\rho + \left(\frac{1+\tau_k}{1+\tau_r}\frac{\kappa}{D}\xi^{-\sigma}\right)^{\frac{1-\gamma}{\gamma}}\left(1-(1-s)\,\xi^{-\sigma}\right) + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right). \tag{D.35}$$

Hence, higher tax rates τ_r and τ_i reduce θ_1, while a higher τ_a coincides with a higher value θ_1. The effect of τ_k is ambiguous, it increases λ but decreases the effective rate of depreciation.

Starting from (D.35) and computing the derivative explicitly, however, shows that the partial derivative $\partial\theta_1/\partial\tau_k = \frac{1}{\sigma}\left((1-(1-s)\,\xi^{-\sigma})\partial\lambda/\partial\tau_k - \frac{1-\tau_i}{(1+\tau_k)^2}\delta\right)$ is positive as long as $(1-(1-s)\,\xi^{-\sigma})\partial\lambda/\partial\tau_k \geq \frac{1-\tau_i}{(1+\tau_k)^2}\delta$. When computing $\partial\lambda/\partial\tau_k = \frac{1-\gamma}{\gamma}\frac{1}{1+\tau_k}\lambda$ from (22), one obtains $\partial\theta_1/\partial\tau_k \geq 0 \Leftrightarrow (1-(1-s)\,\xi^{-\sigma})\frac{1-\gamma}{\gamma}\lambda \geq \frac{1-\tau_i}{1+\tau_k}\delta$, which holds if the rate of physical depreciation δ is sufficiently small.

D.1.1 Coefficient of variation

From (35), volatility increases through higher taxation when θ_1/λ decreases, i.e., $\frac{\partial}{\partial\tau_j}cv^2 \geq 0 \Leftrightarrow \frac{\partial}{\partial\tau_j}(\theta_1/\lambda) \leq 0$. It holds for τ_i and for τ_a. From (D.35),

$$\theta_1/\lambda = \frac{1}{\sigma}\left(\rho + \lambda\left(1-(1-s)\xi^{-\sigma}\right) + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)/\lambda. \tag{D.36}$$

The only partial derivatives for which the sign is ambiguous is for τ_k and τ_r. As τ_r affects only the arrival rate, $\text{sgn}\left[\partial\left(\theta_1/\lambda\right)/\partial\tau_r\right] = \text{sgn}\left[\left(\partial\left(\theta_1/\lambda\right)/\partial\lambda\right)\partial\lambda/\partial\tau_r\right]$. As $\partial\lambda/\partial\tau_r < 0$, the cv increases when $\partial\left(\theta_1/\lambda\right)/\partial\lambda$ is positive. Computing this derivative gives

$$\frac{\partial(\theta_1/\lambda)}{\partial\lambda} = \left(\lambda\frac{\partial}{\partial\lambda}\theta_1 - \theta_1\right)/\lambda^2 > 0 \;\Leftrightarrow\; \lambda\frac{1}{\sigma}(1-(1-s)\,\xi^{-\sigma}) > \theta_1 \;\Leftrightarrow\; 0 > \rho + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a.$$

Since the inequality does not hold for a positive tax of wealth, cv decreases when τ_r increases.

We now compute the partial derivative of (D.36) for τ_k,

$$\begin{aligned}\frac{\partial(\theta_1/\lambda)}{\partial\tau_k} &= \left(\frac{\partial\theta_1}{\partial\tau_k}\lambda - \frac{\partial\lambda}{\partial\tau_k}\theta_1\right)/\lambda^2 = \left[\left(\frac{\partial\theta_1}{\partial\lambda}\frac{\partial\lambda}{\partial\tau_k} - \frac{1-\tau_i}{\left(1+\tau_k\right)^2}\delta\right)\lambda - \frac{\partial\lambda}{\partial\tau_k}\theta_1\right]/\lambda^2 < 0\\ &\Leftrightarrow\; \frac{1}{\sigma}\lambda\left(1-(1-s)\,\xi^{-\sigma}\right)\frac{\partial\lambda}{\partial\tau_k} < \frac{1-\tau_i}{\left(1+\tau_k\right)^2}\delta\lambda + \frac{\partial\lambda}{\partial\tau_k}\theta_1\\ &\Leftrightarrow\; 0 < \frac{1-\tau_i}{\left(1+\tau_k\right)^2}\delta\lambda + \frac{\partial\lambda}{\partial\tau_k}\left[\frac{1}{\sigma}\left(\rho + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)\right],\end{aligned}$$

which holds for any positive tax on wealth.

D.1.2 Measures based on growth rates

From (26), we can study the effect on the variance of output growth rates mainly through the tax effects on the variance of the rental rate of capital, r_t.

$$\frac{\partial Var\left(\frac{1-\tau_i}{1+\tau_k}r\right)}{\partial\tau_j} \geq 0 \;\Leftrightarrow\; \frac{\partial\left(\frac{1-\tau_i}{1+\tau_k}E(r)\lambda\right)}{\partial\tau_j} \geq 0 \;\Leftrightarrow\; \frac{\partial\left(\theta_1\lambda - \ln\left(1-\theta_2\right)\lambda^2\right)}{\partial\tau_j} \geq 0,$$

in which $\theta_1\lambda = \frac{1}{\sigma}\left(\rho + \lambda\left(1-(1-s)\xi^{-\sigma}\right) + \frac{1-\tau_i}{1+\tau_k}\delta + \tau_a\right)\lambda$. The only derivatives for which the sign is ambiguous is for τ_k, $\frac{\partial(\theta_1\lambda)}{\partial\tau_k} = \frac{1}{\sigma}\left(\partial\lambda/\partial\tau_k(1-(1-s)\xi^{-\sigma}) - (1-\tau_i)/(1+\tau_k)^2\delta\right)\lambda + \theta_1$ which remains ambiguous, but for δ sufficiently small is positive.

E Data appendix

E.1 Definitions

For ease of comparison with Mendoza et al. (1994) we retain the variables names based on the definitions in SNA68/ESA79, though the variables are from SNA93/ESA95. In what follows we use the following abbreviations:

Revenue Statistics

1000	Taxes on income, profits and capital gains
1100	Taxes on income, profits and capital gains of individuals
1200	Taxes on income, profits and capital gains of corporations
1300	Unallocable between 1100 and 1200
2000	Social security contributions
2100	Employee's contribution to social security
2200	Employer's contribution to social security
2300	Contribution of self-employed or non-employed to social security
2400	Unallocable as between 2100, 2200 and 2300
3000	Taxes on payroll and workforce
4000	Taxes on property
4100	Recurrent taxes on immovable property
4400	Taxes on financial and capital transactions
5110	General taxes on goods and services
5120	Taxes on specific goods and services
5121	Excise taxes
5122	Profits of fiscal monopolies
5123	Customs and import duties
5125	Taxes on investment goods
5126	Taxes on specific services
5128	Other taxes
5200	Taxes on use of goods and perform activities
5212	Paid by others: motor vehicles
6100	Other taxes paid solely by business

National Accounts

EA	Table 1. Gross Domestic Product: Expenditure Approach
IA	Table 3. Gross Domestic Product: Income Approach
GA	Table 12. Simplified General Government Accounts
HC	Table 13. Simplified Accounts for Households and NPISH and for Corporations

C	Private final consumption expenditure (EA)
G	Government final consumption expenditure (EA)
CoE	Compensation of employees (IA)
GW	Compensation of employees paid by producers of government services (GA)
OS	Operating surplus of the economy; includes statistical discrepancy (IA)
$OSPUE$	Operating surplus and mixed income of private unincorporated enterprises (HC)
PEI	Household's property income (HC)
W	Wages and salaries (IA)

Note that total operating surplus (OS) and operating surplus of private unincorporated enterprises ($OSPUE$) is *net*, that is gross operating surplus minus consumption of fixed capital. Moreover, OS includes the statistical discrepancy.

E.2 Mendoza et. al (1994) tax ratios

The household income tax ratio is equal to personal income tax receipts (1100) divided by household income. Household income comprises operating surplus plus mixed income of the private unincorporated sector ($OSPUE$), property income[18] (PEI), and dependent wage income (W). Given this, the personal income tax reads

$$\tau_h = \frac{1100}{OSPUE + PEI + W}.$$

The labor income tax ratio relates individual labor income tax to total labor costs. Note that $\tau_h W$ allocates household income taxes to labor. All social security charges (2000) and payroll taxes (3000) are also allocated to labor income. Total labor costs are compensation from dependent employment, including employers' social security contributions (2200),

$$LABOR = \frac{\tau_h W + 2000 + 3000}{W + 2200}.$$

The capital tax ratio relates individual capital income (including corporations) and other capital costs to total capital income. Here, $\tau_h(OSPUE + PEI)$ denotes household income taxes related to capital income. The taxes paid directly out of capital income are corporate income taxes (1200), recurrent taxes on immovable property (4100) and taxes on financial and capital transactions (4400),

$$CAPITAL = \frac{\tau_h(OSPUE + PEI) + 1200 + 4100 + 4400}{OS}.$$

The consumption tax ratio is calculated as the sum of general consumption taxes on goods and services (5110) and excise taxes (5121) over the sum of private consumption (C) and government non-wage consumption ($G - GW$) at producer costs,

$$CONS = \frac{5110 + 5121}{C + G - GW - 5110 - 5121}.$$

[18] PEI corresponds to interest, dividends, and investment receipts in SNA93/ESA95.

The effective tax of corporate income relates the taxes paid by corporations (1200) to operating surplus of the corporate sector (obtained as a residual $OS - OSPUE$),

$$CORP = \frac{1200}{OS - OSPUE},$$

which indicates the average tax burden of corporations.